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Article

Optimization of multi-echelon multi-product supply chains using discrete event simulation

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ABSTRACT

This study details the application of a novel multi-echelon, multi-product hybrid supply chain optimization framework known as the Optimizing Known Warehouses (OKW) framework, which consists of mathematical and computer simulation models, to analyze the effects of changes in inventory policies for products delivered by two (2) different case-study manufacturing supply chains. The methods used are the Bayesian optimization algorithm and linear programming to optimize inventory and routing in the supply chains. Furthermore, Arena software was used to develop a discrete event simulation model to compare the current and optimized scenarios of the supply chains. The results show that the optimization framework effectively reduces backorders and increases order fulfillment within the manufacturing supply chains analyzed, and can be easily applied to other Fast-Moving Consumer Goods (FMCG) manufacturing supply chains. For the water bottling supply chain, the optimal inventory policies reduced backorders of the 50cl, 75cl, and 150cl bottled water products by 8.7%, 18.7%, and 34.7%, respectively. For the noodles manufacturing supply chain, backorders for the 70g, 100g, 120g, 180g, and 280g noodle products were reduced by 3.5%, 12%, 20.6%, 7.1%, and 11.9%, respectively. This study provides a procedure for inventory and routing optimization using Bayesian optimization and linear programming, respectively, and a Discrete Event Simulation model for analyzing various supply chain configurations.

1. Introduction

Manufacturing supply chains are networks that distribute products from manufacturers to consumers, through distributors and retailers [1,2]. They convert raw materials from suppliers into finished products within manufacturing plants, then distribute them through distributors and retailers based on consumer demand [3]. To carry out these functions efficiently, supply chains must maintain inventory at various echelons and optimally distribute products between echelons while incurring minimal waste and cost. Improperly managing product demand can lead to various forms of waste within manufacturing supply chains. Therefore, inventory and routing must be continuously adjusted in response to fluctuating demand. Inventory and routing optimization is a common problem among manufacturing supply chains concerned with maximizing profit while minimizing costs. Several researchers have attempted to optimize manufacturing supply chains using hybrid mathematical and simulation methods. Smew et al. [4] provided a framework that supply chain managers could use to assess, with reasonably high accuracy and computational efficiency, the suitability of various production control strategies for their system. Manzini et al. [5] proposed a visually interactive simulation approach as a viable means of making design and management decisions to achieve integrated supply chain optimization. Dai and Zheng [6] aimed to design a closed-loop supply chain network for a multi-period, multi-product, multi-echelon closed-loop supply chain under uncertainty. Memari et al. [7] aimed to perform a simulation study to analyze supply chain system performance under different network configurations, using transportation cost and transportation time as performance criteria. Senyigit and Soylemez [8] investigated the problem of purchasing product components from the most appropriate suppliers, at the most economical prices, producing in the most appropriate quantities, storing the products in the most appropriate quantities, and delivering the products to the customers at the distribution centers, considering the size of the defect between echelons of a supply chain network. Preusser et al. [9]

presented an approach that combines linear programming modeling and discrete event simulation to support the operational decisions of supply chain networks. Sharma et al. [10] aimed to relate product characteristics to optimizing supply chain delivery network design, using cost and service factor performance metrics as decision criteria. Moyaux et al. [11] proposed a coordination technique to reduce fluctuations in orders placed by each company to its suppliers in a supply chain modeled as a multi-agent system. Sitek and Wikarek [12] provided a robust and effective hybrid approach for modeling and optimization of sustainable supply chain management (SCM) and green SCM problems. Fox et al. [13] investigated issues and proposed solutions for developing an agent-oriented software architecture for supply chain management.

Bose and Pekny [14] used model predictive control to control processes and plants in manufacturing supply chains. Brown et al. [15] highlighted the benefits of redesigning Benin's vaccine supply chain. Aliev et al. [16] aimed to develop a fuzzy integrated multi-period and multi-product production and distribution model in a supply chain. Byrne and Heavey [17] modeled and analyzed the effect of information sharing and forecasting on the performance parameters of an actual industrial supply chain consisting of small-to-medium enterprises. Minegishi and Thiel [18] aimed to show how system dynamics can improve understanding of the complex logistical behavior of an integrated food industry. Selim et al. [19] developed a supply chain distribution network design model that selects the optimum numbers, locations, and capacity levels of plants and warehouses to deliver products to retailers at the lowest cost, while satisfying the desired service level for retailers. Taktak et al. [20] developed a computer-assisted performance analysis and optimization model to help small- and medium-sized enterprise managers conduct simulation projects.

Kiesmüller et al. [21] presented a dual supply chain model that accounts for the replenishment cycle involving not only the physical distribution of goods but also product manufacturing. Disney and Towill [22] considered the performance of a production or distribution-scheduling algorithm termed Automatic Pipeline, Inventory and Order Based Production Control System (APIOBPCS) embedded within a vendor-managed inventory supply chain where the demand profile changes significantly over time. Almeder et al. [23] presented a general framework to support the operational decisions for supply chain networks using a combination of an optimization model and discrete event simulation. Truong and Azadivar [24] developed a hybrid optimization approach to address the supply chain configuration design problems. Van der Vorst et al. [25] presented a method for modeling the dynamic behavior of food supply chains and evaluating alternative supply chain designs using discrete event simulation. Lee and Kim [26] proposed a hybrid approach that combines analytic and simulation methods to solve production-distribution problems in supply chains.

Gupta and Maranas [27] used a framework for mid-term, multi-site supply chain planning under demand uncertainty to safeguard against inventory depletion at production sites and excessive shortages at the customer. Dabbene et al. [28] aimed to present an approach to optimizing fresh-food supply chains that manage a trade-off between logistics costs and indices that measure food quality, such as ripeness, microbial charge, or internal temperature. Acar et al. [29] developed a decision support framework for a global manufacturer of specialty chemicals to study the relative impact of demand, supply and lead-time uncertainties on cost and customer service performance. Shukla et al. [30] proposed a hybrid approach for resolving the complexities involved in the dynamic interaction among multiple facilities and locations in a supply chain. Ko et al. [31] proposed a hybrid optimization/simulation approach to design a distribution network for third-party logistics, considering warehouse performance. Skapinyecz [32] reviewed recent developments and future approaches in logistics system optimization using Deep Learning (DL) and Discrete-Event Simulation (DES). Gebreabzgi et al. [33] used discrete event simulation to demonstrate the value of simulation optimization for supply chains in low-resource environments where operational restrictions and demand fluctuations present difficulties. Kogler and Maxera [34] surveyed the literature on supply chain analyses that combined machine learning techniques similar to Bayesian optimization with discrete event simulation modeling. Li et al. [35] introduced Bayesian optimization for cold chain warehouse demand forecasting to reduce resource waste and efficiency bottlenecks in conventional cold storage management models.

From the foregoing, there is a need to develop systems and frameworks to solve the inventory and routing problem of multi-echelon, multi-product supply chains to reduce supply chain costs while increasing efficiency, productivity, and profitability. Therefore, the optimization process must traverse the entire supply chain network while maximizing performance across all echelons [36]. Although there is extensive research on managing supply chains for numerous products, studies on the Fast-Moving Consumer Goods (FMCG) industry supply chain remain limited [37]. This study aims to propose a framework for studying, analyzing, and managing multi-echelon, multi-product supply chains and to provide a generic hybrid supply chain optimization framework for solving the multi-echelon, multi-product inventory and routing management problem in the Fast-Moving Consumer Goods (FMCG) industry. Moreover, this research adds to existing studies on multi-echelon, multi-product supply chain management. The study provides a framework for managing the distribution aspect of multi-echelon, multi-product supply chains to reduce supply chain costs from backorders while increasing efficiency, productivity, and profitability.

2. Methodology

This section discusses the tools, software, methods, and equations used in this study, with emphasis on how they were used to model, analyze, and optimize the supply chain. The Optimizing Known Warehouses (OKW) framework has three (3) constituents, namely: the OKW Mathematical Inventory Optimization Model, the OKW Mathematical Routing Optimization Model, and the OKW Discrete Event Simulation Model. The Python programming language was used to develop the code for the Bayesian optimization algorithm. The version of Python used is version 3.10.5. The Bayesian optimization Python library was used to provide an optimal inventory policy for warehouses within the manufacturing and distribution echelons of the supply chains, using the OKW Mathematical Inventory Optimization Model. Also, the PuLP library, which is a Python linear programming modeler, was used to find the most cost-efficient route for distributing products in the supply chains using the OKW Mathematical Routing Optimization Model. Minitab statistics software version 19.2.0 was also used as a tool for analyzing data. Minitab is a statistical software package that automates calculations and graph creation, allowing the user to focus more on data analysis and interpretation of results. Moreover, Arena simulation software is a discrete-event simulation and automation software owned by Rockwell Automation, Inc. It uses the SIMAN processor and simulation language to model manufacturing and service systems. Arena simulation software is designed to analyze the impact of changes involving significant and complex redesigns associated with supply chain, manufacturing, processes, logistics, distribution and warehousing, and service systems [38]. This study uses Arena version 14.00.00000. The Arena Process Analyzer (PAN) is a tool used to conduct parametric analysis of Arena models. In Arena jargon, input parameters are referred to as controls, the resultant performance measures are referred to as responses, the collection of controls and responses for a given set of runs is called a scenario, and a collection of scenarios is referred to as an Arena project. According to Altiok and Melamed [39], the Process Analyzer conducts parametric analysis by allowing the modeler to create, run, and compare simulated scenarios, and thus observe the effect of prescribed controls on prescribed responses. Therefore, the Arena simulation software was used to compare the current and optimized configurations of the manufacturing supply chains through the OKW Discrete Event Simulation model.

2.1 Description of the multi-echelon multi-product supply chain

Consider a multi-product, multi-echelon supply chain consisting of a distribution center, manufacturing plant, and supplier. The manufacturing plant contains a production facility that handles production and interacts with two buffers: an input buffer, which stores incoming raw materials, and an output buffer, or factory warehouse, which stores outgoing finished products. Figure 1 schematically depicts the multi-echelon, multi-product supply chain system.

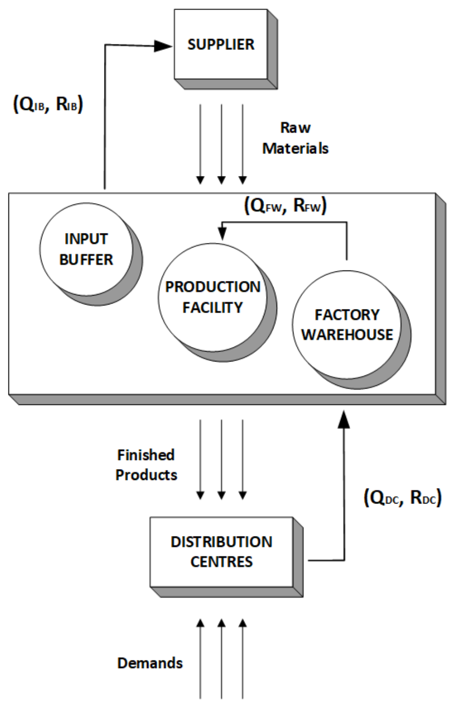

Figure 1. Multi-echelon multi-product supply chain system

From Figure 1, retailer demand for various product types enters the system at the distribution center echelon downstream in the supply chain. Distributors fulfill retailer demand, and once the inventory position of a specific product type at the distributor echelon falls to R_{DC}, the distributor orders a replenishment quantity, Q_{DC}, of the product from the output buffer or factory warehouse. The inventory position equals available inventory plus outstanding orders minus backorders for the product. The manufacturing plant output buffer replenishes the distribution center and is replenished by the production facility when its stock for any product handled by the supply chain drops below R_{FW}. The production facility manufactures the products handled by the supply chain and receives raw materials from the manufacturing plant input buffer. The manufacturing plant input buffer contains raw materials required for production and orders a replenishment quantity, Q_{IB}, from the supplier located upstream of the supply chain when its raw material stock drops below R_{IB}.

2.2 The OKW mathematical inventory optimization model

The objective function of the OKW inventory optimization model, considering all products handled by the manufacturing supply chain, multiple factory warehouses, and multiple distribution warehouses, is of the following form [40]:

$$\text{Profit} = \sum_{h=1}^{o}\left(\sum_{m=1}^{k}\left(\sum_{j=1}^{w} N_j R_j - n_i(p_c + c_h) - \sum_{j=1}^{w} I_j H_j - \sum_{j=1}^{w} O_j V_j - \sum_{j=1}^{w} D_j F_j\right)_m\right)_h \tag{1}$$

Subject to:

$$\sum_{m=1}^{k}\left(n_i(p_c + c_h) + \sum_{j=1}^{w} I_j H_j + \sum_{j=1}^{w} O_j V_j + \sum_{j=1}^{w} D_j F_j\right)_m \leq B \tag{2}$$

$$\sum_{m=1}^{k}(n_i)_m \leq \sum_{m=1}^{k}(C_f)_m \tag{3}$$

$$n_i > 0 \tag{4}$$

$$0 < N_j \leq C_j \text{ for } j = 1, 2, \cdots, w \tag{5}$$

$$0 < I_j \leq C_j \text{ for } j = 1, 2, \cdots, w \tag{6}$$

$$0 < O_j \leq C_j \text{ for } j = 1, 2, \cdots, w \tag{7}$$

where o is the total number of products handled by the supply chain; k is the total number of factory warehouses in the supply chain; w is the total number of distribution warehouses supplied by a factory warehouse m; N_j is the number of units sold by distribution warehouse j; R_j is the revenue per unit sold by distribution warehouse j; n_i is the factory warehouse inventory level; p_c is the factory production cost per unit; c_h is the factory warehouse holding cost per unit; I_j is the inventory level at distribution warehouse j; H_j is the holding cost per unit at distribution warehouse j; O_j is the number of items ordered by distribution warehouse j; V_j is the variable order cost at distribution warehouse j; D_j is the number of orders made by distribution warehouse j; F_j is the fixed order cost per order at distribution warehouse j; B is the factory budget for production and distribution of product h; C_j is the capacity of the distribution warehouse j; C_f is the factory warehouse capacity.

The OKW inventory optimization model is a linear model having a linear objective function and linear constraints. The model considers various costs, including production, holding, variable order, and fixed order costs. The optimization objective is to maximize profit. The maximum profit value was obtained using the Bayesian optimization algorithm implemented in Python through the Bayesian-optimization Python library [41]. According to Garnett [42], the Bayesian optimization algorithm is a sequential optimization algorithm for global optimization of objective functions that builds a probability model of the objective function and uses it to select hyperparameters to evaluate in the true objective function to obtain the optimum value of the function. Therefore, when searching for the optimal value of the objective function and its parameters, the algorithm balances exploring unsearched areas of the feasible region (exploration) with exploiting promising neighborhoods (exploitation), based on its knowledge of the target function.

2.3 The OKW mathematical routing optimization model

Routing optimization is the process of determining the most cost-efficient route for distributing products of the supply chain. For routing optimization using the OKW routing optimization model, the objective is to minimize the total cost in the following form [40]:

$$\text{Cost} = \sum_{h=1}^{o}\left(k_f\left(\sum_{j=1}^{w} b_{cj} y_j + \sum_{j=1}^{w} e_{cj} y_j + \sum_{j=1}^{w} l_{cj} y_j + \sum_{j=1}^{w} p_{cj} y_j + \sum_{j=1}^{w} i_{cj} y_j + \sum_{i=1}^{n_t}\sum_{j=1}^{w} c_{ij} x_{ij}\right)\right)_h \tag{8}$$

Subject to:

$$1.05 \leq k_f \leq 1.10 \tag{9}$$

$$\sum_{j=1}^{w} x_{ij} = d_i \text{ for } i = 1, 2, \cdots, n_t \tag{10}$$

$$\sum_{i=1}^{n_t} x_{ij} \leq M_j y_j \text{ for } j = 1, 2, \cdots, w \tag{11}$$

$$0 \leq x_{ij} \leq d_i y_j \text{ for } i = 1, 2, \cdots, n_t; \text{ for } j = 1, 2, \cdots, w \tag{12}$$

$$y_j \in \{0, 1\} \text{ for } j = 1, 2, \cdots, w \tag{13}$$

where o is the total number of products handled by the supply chain; k_f is a contingency factor; w is the total number of distribution warehouses; b_{cj} is the building or rental cost of the distribution warehouse j; y_j is a binary variable that indicates whether distribution warehouse j is available; e_{cj} is the equipment operating and maintenance cost of the distribution warehouse j; l_{cj} is the labor or personnel cost of the distribution warehouse j; p_{cj} is the power and electricity cost of the distribution warehouse j; i_{cj} is the insurance cost of the distribution warehouse j; n_t is the total number of sub-distribution warehouses; c_{ij} is the transportation cost of moving from distribution warehouse j to sub-distribution warehouse i; x_{ij} is the quantity of products moved from distribution warehouse j to sub-distribution warehouse i; d_i is the demand at the sub-distribution warehouse i; M_j is the maximum quantity of products that can be handled at the distribution warehouse J.

The OKW routing optimization model is a linear programming model having a linear objective function and linear constraints. The model considers various costs, including building or rental costs, equipment operating and maintenance costs, labor or personnel costs, power and electricity costs, insurance costs, transportation costs, as well as contingency costs. The crux of routing optimization is determining the appropriate routes and the quantity of products to ship from distribution warehouses to sub-distribution warehouses to minimize total cost, thereby aiding profit maximization. Therefore, the PuLP linear programming library was used to minimize the total cost in the objective function.

2.4 The OKW Arena discrete event simulation model

The supply chains being modeled in this work are multi-echelon supply chains consisting of supplier, factory, and distribution center echelons. The OKW Arena discrete event simulation supply chain model utilizes Arena simulation software developed by Rockwell Automation. The model also uses installation stock policies, which are policies based only on inventory position data, which consists of available inventory, outstanding orders, and backorders. This contrasts with echelon stock policies, which require additional information in the form of inventory positions at the current installation and at all downstream echelons. The following assumptions were made while developing the Arena model of the supply chain system:

i. There is no aggregation of various types of products in orders.
ii. The system uses installation stock policies, i.e., replenishment of stock is based only on information at a particular echelon.
iii. The distribution center, factory warehouse, and manufacturing plant input buffer utilize the order-quantity/reorder-point (Q, R) inventory control policy.
iv. Production of a particular item continues until the production order is fulfilled i.e. production of a particular type of product is not interrupted by production of other products.
v. Raw materials for all types of products are always available from the supplier.
vi. Setup delays of the manufacturing plant are zero.
vii. Processing times of the individual stages of production for each type of product are combined into a single processing time.
viii. There is no overtaking of orders; orders are received in the order they were placed.
ix. In situations where the distribution center or manufacturing plant is out of stock, and unsatisfied portions of orders are backordered, fulfillment of orders is deferred until the full order becomes available.
x. Products are not ordered and shipped among members of a supply chain echelon, i.e., products are not ordered and shipped between a distribution center and another distribution center in the same echelon.

Figures 2-5 show the discrete event simulation models for the distribution warehouse, factory warehouse, output buffer, input buffer, and supplier sections of the manufacturing supply chain, respectively [40].

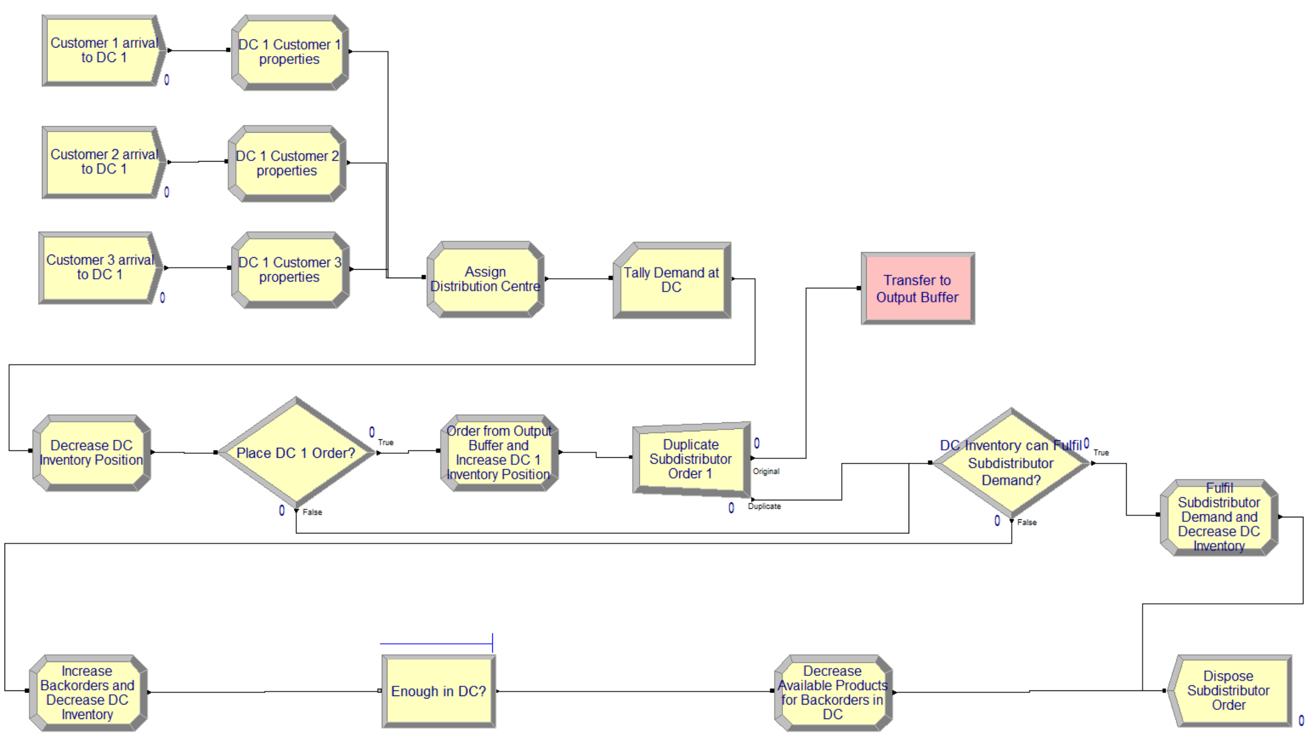

Figure 2. Arena discrete event simulation model of the distribution warehouse section of the manufacturing supply chain

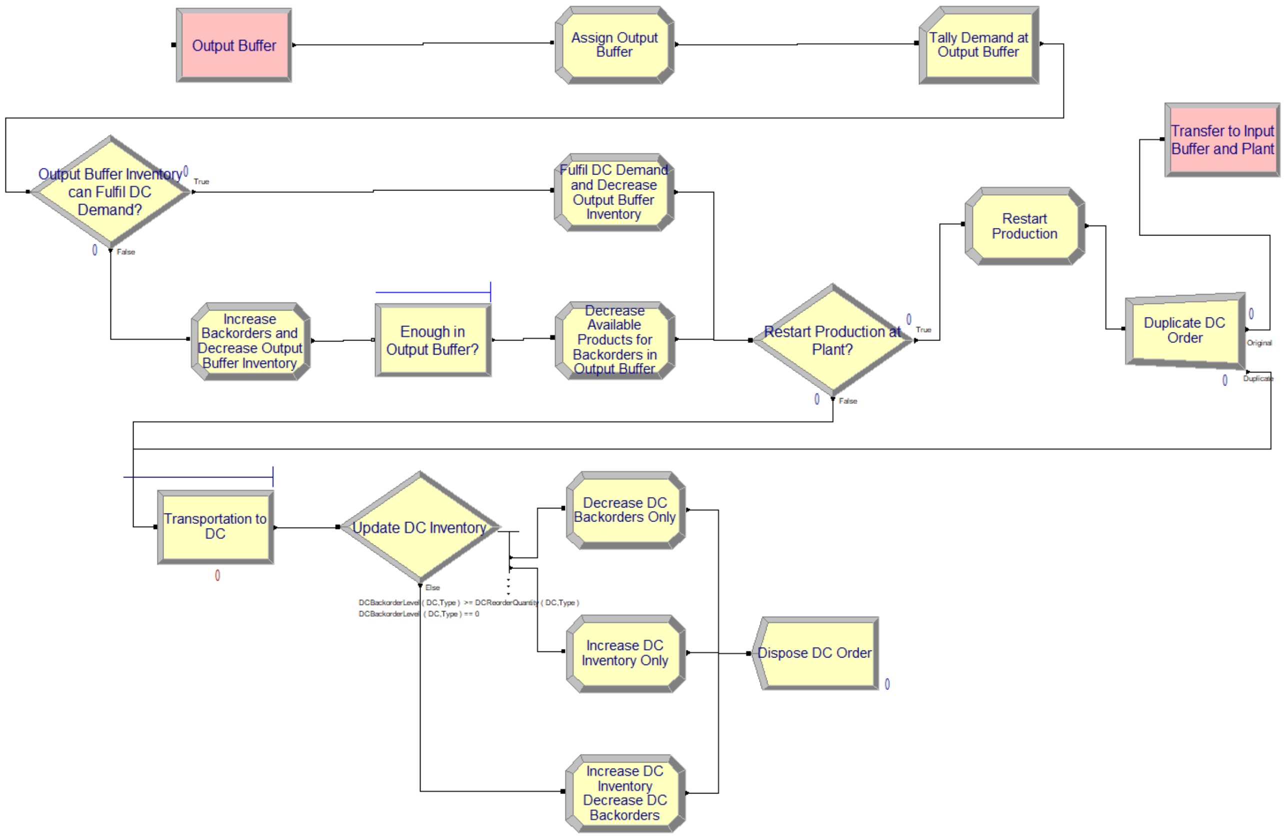

Figure 3. Arena discrete event simulation model of the factory warehouse section of the manufacturing supply chain

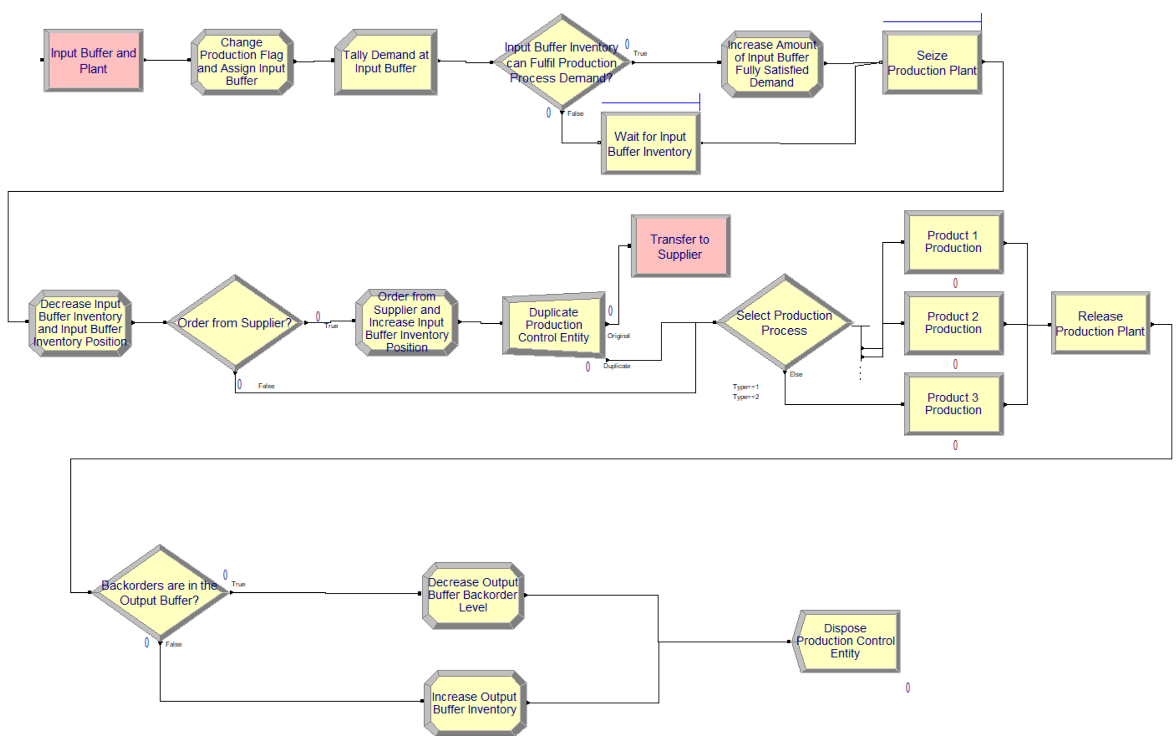

Figure 4. Arena discrete event simulation model of the input buffer section of the manufacturing supply chain

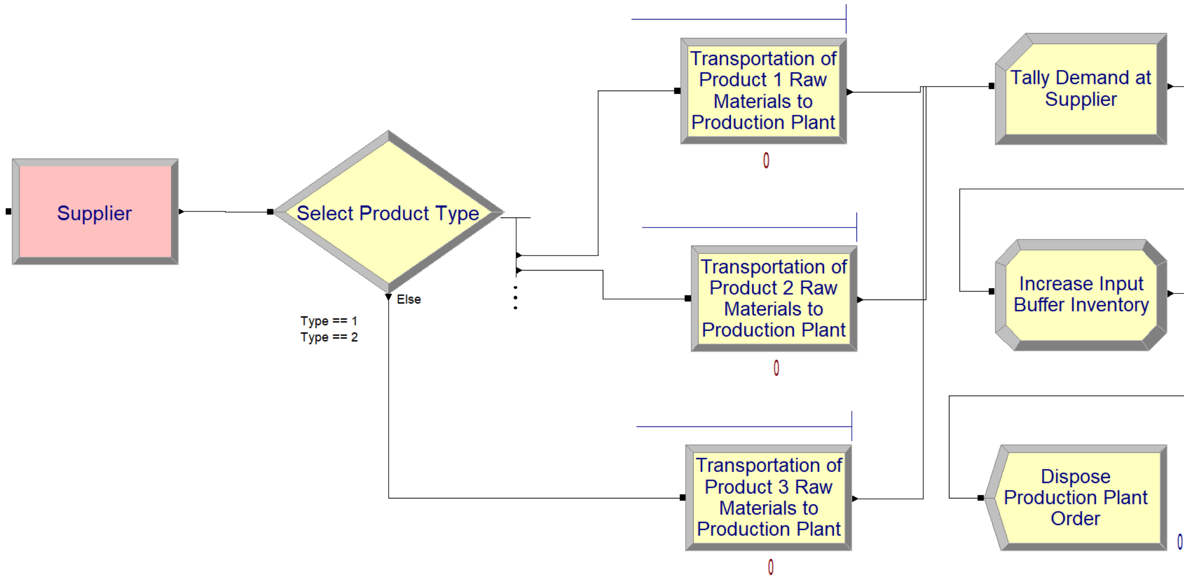

Figure 5. Arena discrete event simulation model of the supplier section of the manufacturing supply chain

2.5 Goodness of the Arena model

Because a simulation model is a simplification of the system under study, it will rarely replicate system characteristics precisely. Therefore, one of the fundamental goals of creating simulation models is to develop an acceptably correct model. According to Altiok and Melamed [39], model goodness is relative, rather than absolute, and it refers to an operational notion that requires the model to produce sufficiently accurate performance predictions for the system under study. Model verification and validation refer to a group of methods and techniques which are used to assess the goodness of the simulation model.

Verification of the Arena model: Verification of the Arena model is necessary because it helps in ensuring that the model is correctly constructed, conforms to its design specifications, and does what it was developed to do. Therefore, verification of the Arena model assesses the correctness of the computer simulation program developed in Arena by inspecting the code, conducting test runs, and performing consistency checks on output statistics. Although model verification is carried out predominantly by inspecting and comparing the model code with the model specifications, several methods and techniques exist for verifying simulation models. According to Altiok and Melamed [39], verifying an Arena model consists mainly of inspecting and walking through the simulation program logic. Verification also includes inspecting code printouts, graphics, and sample path routes during simulation test runs to verify that the underlying program logic is accurate. Again, the process includes basic consistency checks, including sanity checks, as well as more sophisticated checks of theoretical relationships among the statistics to be predicted.

Validation of the Arena model: Model validation refers to the process of checking a model's credibility by comparing selected performance measures predicted by the model with their empirical equivalents measured from the real-life system under study. Basically, validation of the Arena model examines how well the model fits empirical data. Empirical data refers to measurements of the real-life system being modeled. A good model fit implies that a set of important performance measures, predicted by the model, reasonably match or agree with their observed counterparts in the real-life system. Therefore, validation of the Arena model assesses the pragmatism of the modeling assumptions by comparing model performance metrics or predictions from model test runs with their counterparts in the real-life system under study to determine the goodness-of-fit of the data points.

To carry out model validation, a sample $\{\overline{M}_1, \dots, \overline{M}_N\}$ of a particular observed output metric is collected from the real-life system under study over days, $i = 1, \dots, N$. Also, a sample $\widehat{M}_1, \dots, \widehat{M}_N$ of the estimated output metric is collected from runs of the simulation model over an equal number of days, $i = 1, \dots, N$ as that of the real-life system under study. Hypothesis testing compares the two groups of the output metric to determine whether they are statistically similar or different. This comparison is carried out by determining confidence intervals or by conducting a two-sample t-test. Therefore, a sample is defined as the differences.

$$D_i = \widehat{M}_i - \overline{M}_i, \text{for } i = 1, \dots, N \tag{14}$$

which are approximately normally distributed with a mean μ_D and variance S_D^2. The hypothesis testing then assumes the form:

$$\begin{cases} H_0: \mu_D = 0 \\ H_1: \mu_D \neq 0 \end{cases} \tag{15}$$

where H_0 is the null hypothesis, and H_1 is the alternative hypothesis.

Under the null hypothesis, H_0: $\mu_D = 0$, for N – 1 degrees of freedom, at α significance level, and (1 – α) % confidence level, the t statistic, obtained from Johnson et al. [43] as:

$$t = \frac{\overline{D} - \mu_D}{S_D/\sqrt{N}} \tag{16}$$

where $\overline{D}$ and S_D are the sample mean and sample standard deviation of the sample $\{D_1, \ldots, D_N\}$, respectively. If $t < -t_{\alpha/2,N-1}$ or $t > t_{\alpha/2,N-1}$, then the null hypothesis that $\mu_D = 0$ can be rejected at the α significance level, indicating that the model is invalid. If, however, $-t_{\alpha/2,N-1} < t < t_{\alpha/2,N-1}$, then the null hypothesis that $\mu_D = 0$ cannot be rejected at the α significance level, suggesting that the model is valid. The validity of the model can also be determined from the confidence interval obtained from Johnson et al. [43] as:

$$\overline{D} - t_{\alpha/2,N-1}\frac{S_D}{\sqrt{N}} \leq \mu_D \leq \overline{D} + t_{\alpha/2,N-1}\frac{S_D}{\sqrt{N}} \tag{17}$$

If the confidence interval in equation (17) contains 0, then H_0 cannot be rejected at the α significance level, indicating that the hypothesis test supports model validity. If, however, the confidence interval does not contain 0, the test suggests that the model is invalid. Conducting the two-sample t-test in Minitab and evaluating the resulting P-value can also support the hypothesis test results. If the P-value is greater than 0.05, you cannot reject the null hypothesis, suggesting the model is valid. If, however, the P-value is less than 0.05, the alternative hypothesis is favored, and the null hypothesis can be rejected, suggesting that the model is not valid.

2.6 Output analysis

The Arena simulation model was run, and the output statistics and simulation results from various replications were computed and analyzed. Experimentation with different input parameters, operating scenarios, and alternative system designs forms the core of output analysis in Arena and will help optimize the supply chain. Output analysis can be conducted by utilizing Arena tools such as the standard Arena output report and the Arena Process Analyzer (PAN). PAN is the predominant tool used for parametric analysis of Arena models. According to Altiok and Melamed [39], parametric analysis can be defined as the process of running a model several times while varying the input parameter sets for each run, and then comparing the resultant performance measures. This helps understand how parameter changes affect system behavior by identifying the optimal system configuration with respect to one or more performance measures.

3. Results and discussion

The methods and equations from the previous section were applied to optimize the inventory and routing of two (2) different manufacturing supply chains. The supply chains are a bottled water supply chain and a noodle manufacturing supply chain, both Fast-Moving Consumer Goods (FMCG) manufacturing supply chains.

3.1 Case study 1: water bottling supply chain

The bottled water supply chain consists of raw material suppliers, a manufacturing plant, two distribution centers, and four sub-distribution centers. The products handled by the supply chain include 50cl, 75cl, and 150cl bottled water.

Inventory optimization of the bottled water supply chain: The mathematical inventory optimization model in Section 2.2 was applied to optimize inventory in the factory warehouse and two distribution warehouses. The supply chain was optimized by considering only one product at a time. Figures 6-8 show plots of profit from various iterations for the 50cl, 75cl, and 150cl bottled water products, respectively. The plots were created using the Matplotlib Python library and show the results of inventory optimization using the Bayesian optimization algorithm. The inventory optimizations were conducted for up to 150 iterations. The Bayesian optimization algorithm searches the space of possible maximum profits from each iteration to find the optimal profit. The figures plot profit versus iterations and show the maximum profit obtained for each product, along with the iteration at which it was achieved. From the figures, the optimizer explores the feasible solution space and quickly identifies solutions that yield substantially higher profits than the initial random evaluations. As the optimization progresses, the surrogate model learns the relationship between decision variables and profit, enabling the optimizer to focus its search on promising regions while occasionally exploring uncertain areas. The observed convergence pattern illustrates that Bayesian Optimization can efficiently maximize profit while significantly reducing the number of objective function evaluations required compared to traditional optimization approaches.

Routing optimization of the bottled water supply chain: The routing optimization model in Section 2.3 was applied to determine the most cost-effective routes for distributing products between the two distribution centers and four sub-distribution centers of the water bottling supply chain. The supply chain was optimized by considering only one product at a time.

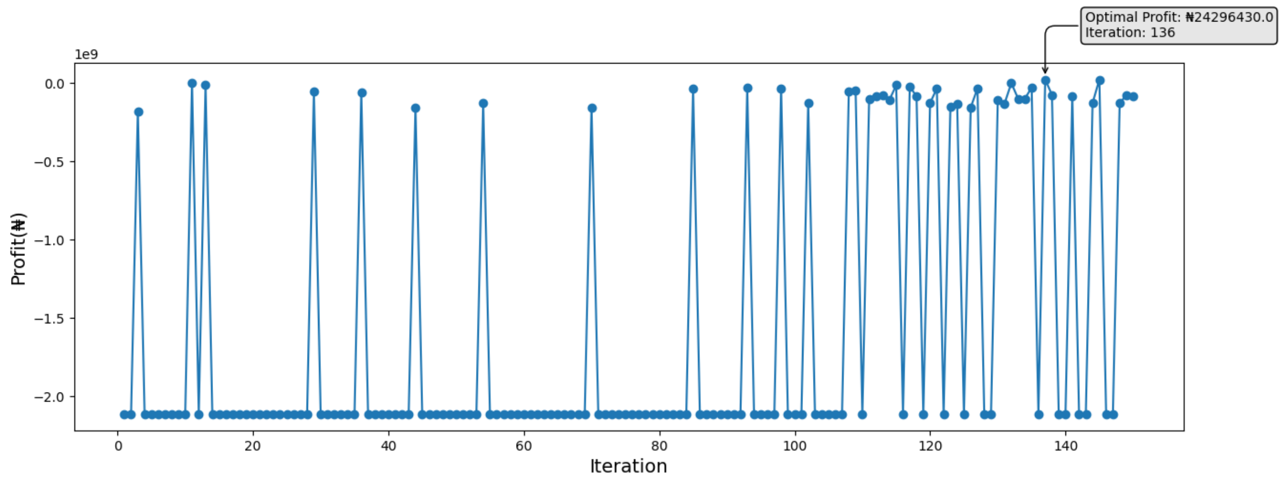

Figure 6. Plot of Profit against Iteration for 50cl bottled water

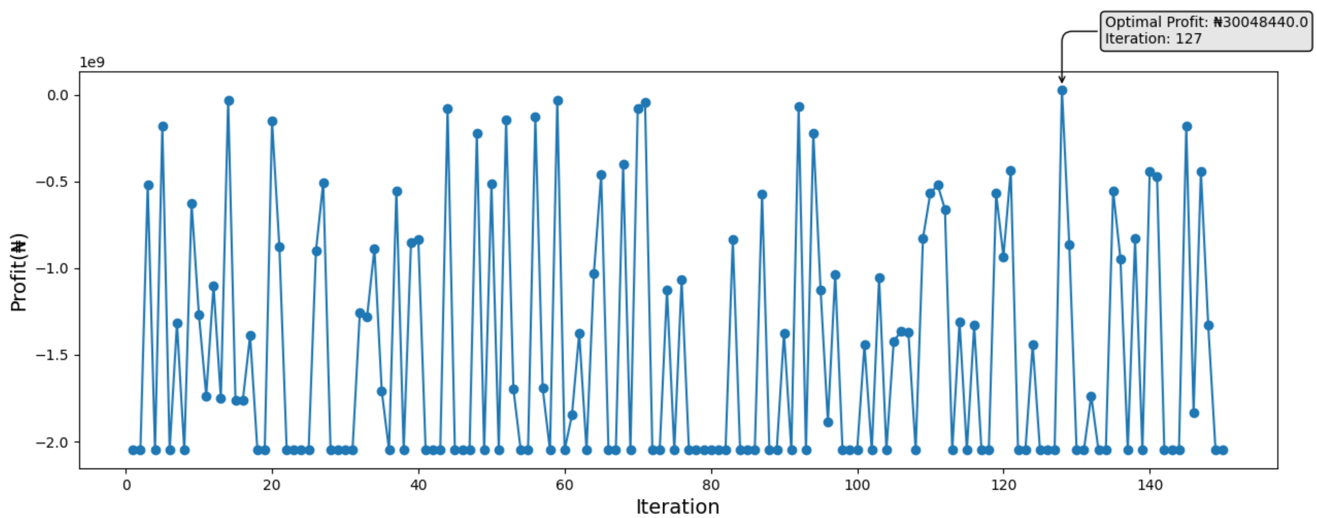

Figure 7. Plot of Profit against Iteration for 75cl bottled water

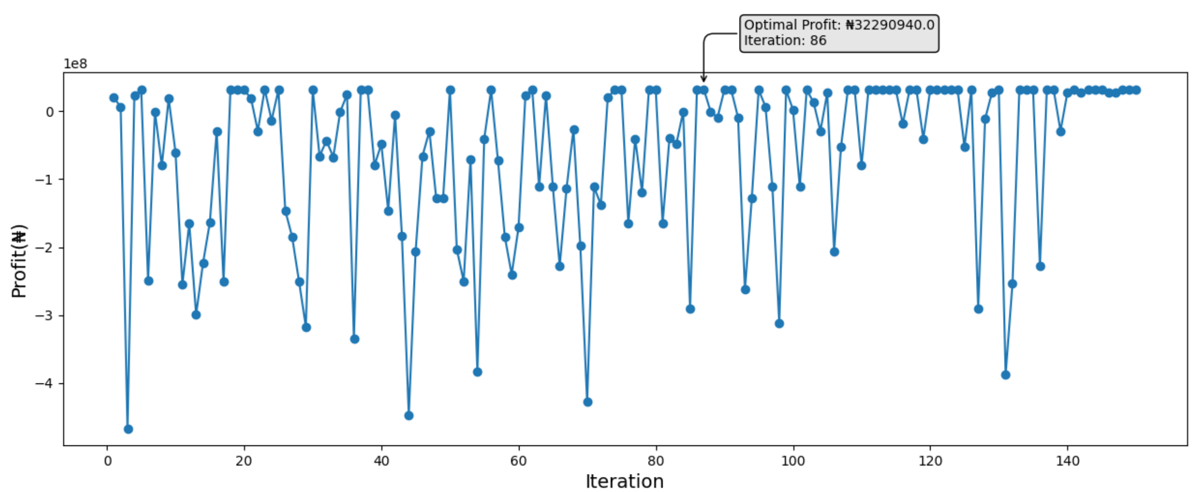

Figure 8. Plot of Profit against Iteration for 150cl bottled water

Figures 9-11 are bar charts that show, respectively, the quantity of 50cl, 75cl, and 150cl bottled water packs each distribution warehouse is expected to supply to each sub-distribution warehouse in order to achieve minimal monthly total costs of distribution warehouse operation and cost of transportation of products to sub-distribution warehouses. The routing optimization results from the data-driven model show that the proposed framework enables supply chain analysts to assess alternative inventory policies, routing strategies, and supply chain configurations, supporting informed managerial decision-making that minimizes costs, improves operational efficiency, reduces stock shortages, and enhances customer satisfaction. The results show that distribution warehouse 2 better meets the demands of the water bottling supply chain because it has more capacity and is closer to demand points. The optimization model selects warehouse allocations and routing decisions that collectively minimize the objective function by balancing trade-offs between inventory, warehousing, and transportation costs. Rather than minimizing each cost component separately, the model identifies the allocation and routing combination that yields the lowest total supply chain cost while satisfying all operational constraints. This integrated approach improves supply chain performance and minimizes the objective function by lowering operating costs and improving resource utilization.

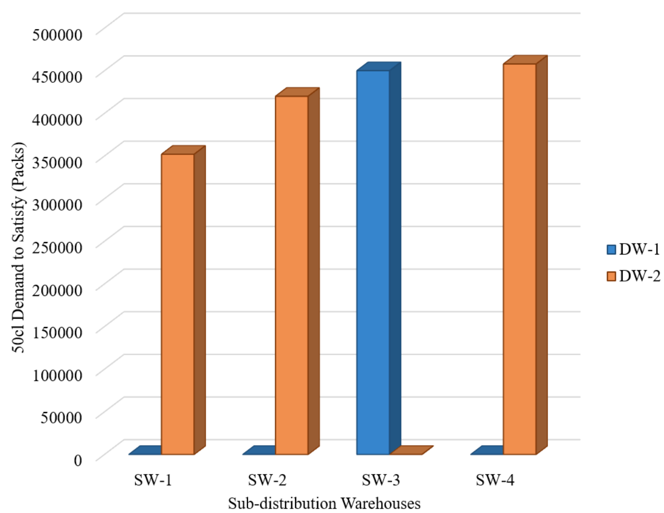

Figure 9. Quantity of 50cl bottled water packs that each distribution warehouse (DW) is expected to supply to each sub-distribution warehouse (SW)

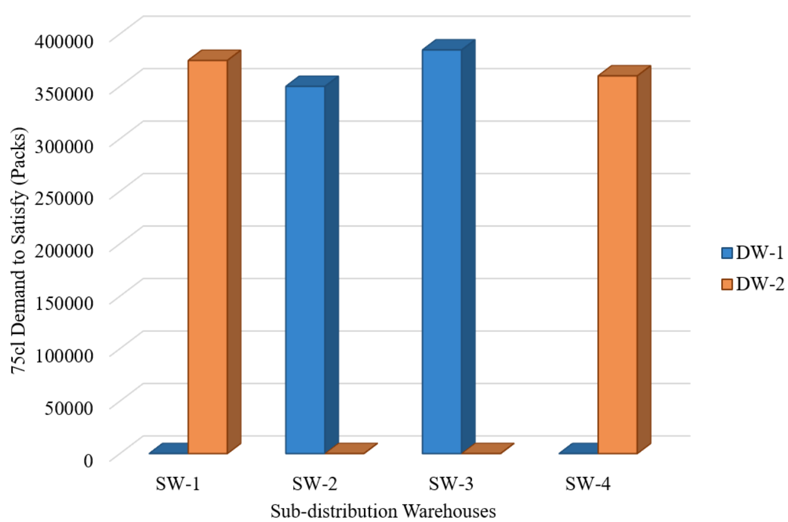

Figure 10. Quantity of 75cl bottled water packs that each distribution warehouse (DW) is expected to supply to each sub-distribution warehouse (SW)

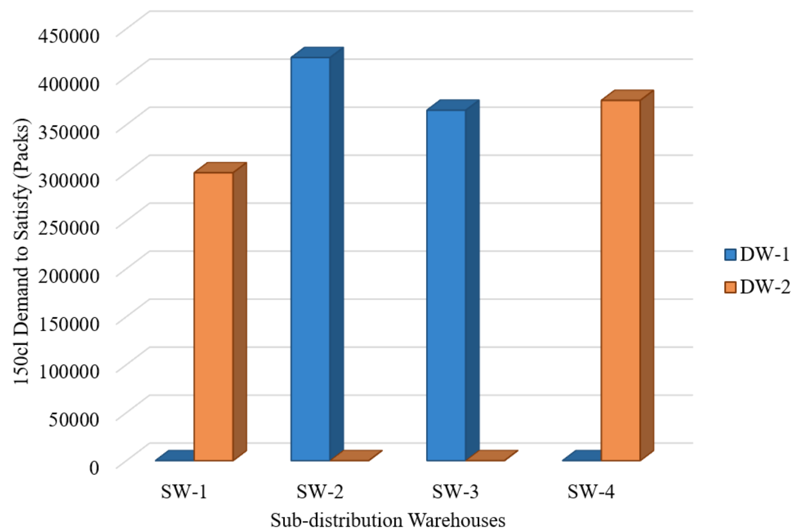

Figure 11. Quantity of 150cl bottled water packs that each distribution warehouse (DW) is expected to supply to each sub-distribution warehouse (SW)

Discrete event simulation of the bottled water supply chain: The discrete event simulation model was applied to bottled water supply chain data to compare average backorders in the current state with those in the optimized state. The discrete event simulation model was also validated to ascertain its effectiveness in analyzing the bottled water supply chain using the equations in section 2.5. To verify the Arena model, various verification activities were conducted such as conducting an inspection and walk-through of the simulation program logic, inspection of code printouts, graphics and sample path routes while performing simulation test runs, to verify that the underlying program logic is accurate, as well as carrying out basic consistency checks, including sanity checks, as well as more sophisticated checks of theoretical relationships among statistics to be predicted. To validate the Arena model, ten (10) randomly selected observations of processing time for the 50cl bottled water were compared with the processing time from ten (10) replications of the Arena model using a two-sample t-test in Minitab statistical software. Table 1 shows the observed and simulated processing time data for 50cl bottled water. The two-sample t-test at the 0.05 significance level yielded a p-value of 0.685, which is greater than 0.05; therefore, the null hypothesis cannot be rejected, suggesting that the model is valid.

Table 1. Data on observed processing time and simulated processing time for 50cl bottled water

S/No.	Observed Processing Time (Seconds)	Simulated Processing Time (Seconds)
1	39	41.54
2	38	40.69
3	40	40.70
4	37	36.83
5	38	35.90
6	37	40.40
7	39	37.40
8	40	41.23
9	36	30.99
10	37	31.06

The Arena model was run, and the current inventory policy of the bottled water supply chain was compared with the optimized inventory policy. Figure 12 compares the average backorders for the three (3) products handled by the supply chain under the current and optimized inventory policies. Backorders are orders that cannot be fulfilled immediately due to insufficient inventory. From Figure 12, the optimized inventory policy reduced backorders of the 50cl, 75cl, and 150cl bottled water by 8.7%, 18.7%, and 34.7%, respectively.

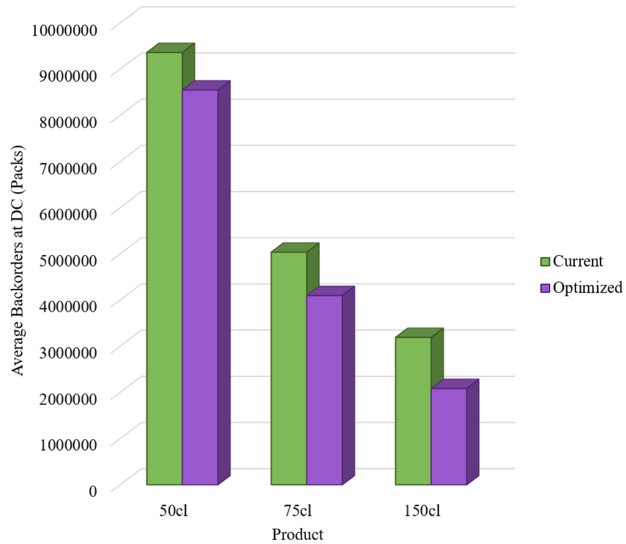

Figure 12. Comparison of the average backorders for the three products of the bottled water supply chain, considering the current inventory policy and the optimized inventory policy

3.2 Case study 2: Noodles manufacturing supply chain

The noodles manufacturing supply chain consists of raw material suppliers, a manufacturing plant, three distribution centers, and five sub-distribution centers. The supply chain handles 70g, 100g, 120g, 180g, and 280g noodle products. Moving upstream of the noodle manufacturing supply chain, demand data were obtained from the sub-distributors.

Inventory optimization of the noodle manufacturing supply chain: The mathematical inventory optimization model in Section 2.2 was applied to optimize inventory in the factory warehouse and three distribution warehouses. The supply chain was optimized by considering only one product at a time. Figures 13-17 show the profit plots from various iterations for the 70g, 100g, 120g, 180g, and 280g noodle products, respectively. The plots were created using the Matplotlib Python library and show the results of inventory optimization using the Bayesian optimization algorithm. The inventory optimizations were conducted for up to 150 iterations. The Bayesian optimization algorithm searches the space of possible maximum profit from each iteration to find the optimal profit. The figures plot profit versus iterations and show the maximum profit obtained for each product's inventory optimization, along with the iterations on which those profits were achieved. From the figures, the plots are nearly horizontal, with sharp initial rises and little to no improvement over many consecutive iterations. Therefore, profit increased rapidly during the early iterations as the algorithm explored the search space and identified promising inventory policies. This indicates that the optimizer converged to the global optimum or a near-optimal solution, and additional iterations produced diminishing returns because most promising regions have already been explored. This convergence pattern demonstrates the efficiency of Bayesian Optimization in maximizing profit while requiring relatively few objective function evaluations compared with traditional optimization techniques.

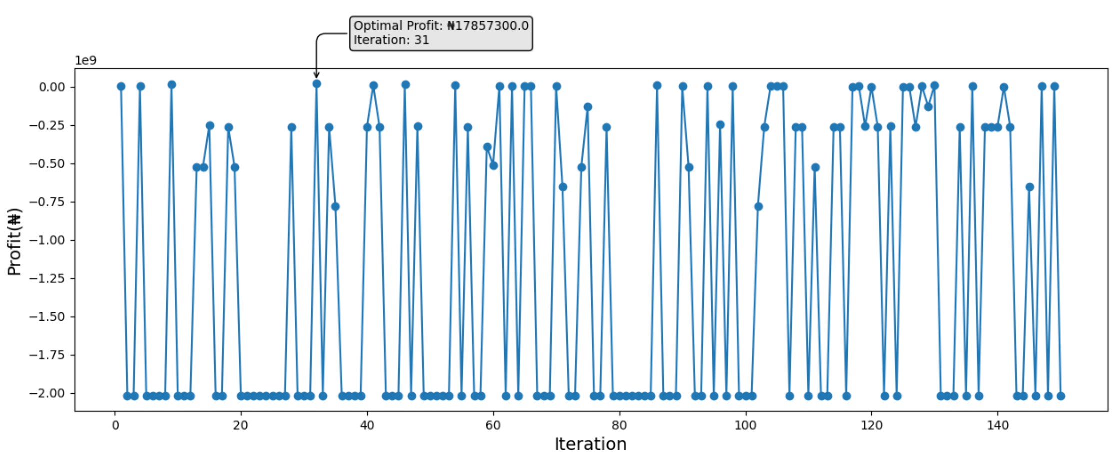

Figure 13. Plot of Profit against Iteration for the 70g noodles product

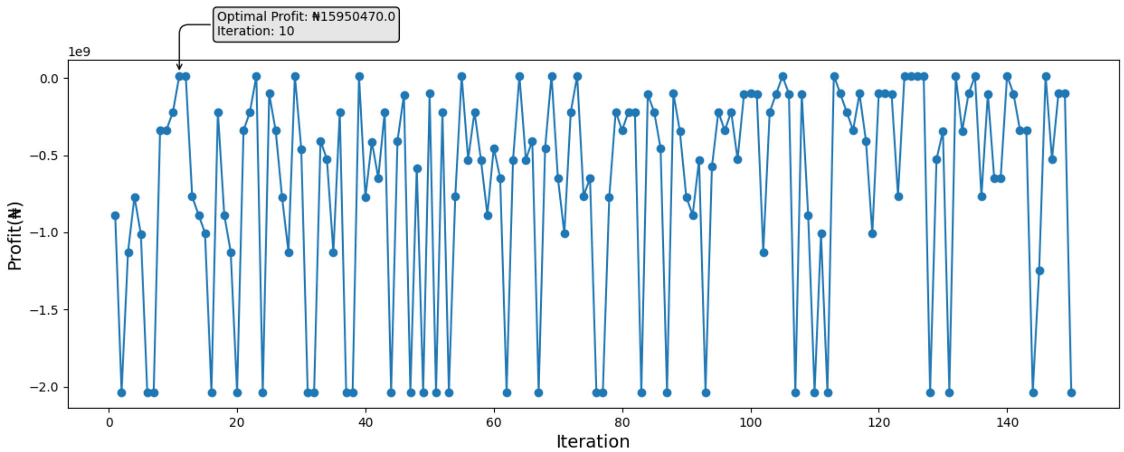

Figure 14. Plot of Profit against Iteration for the 100g noodles product

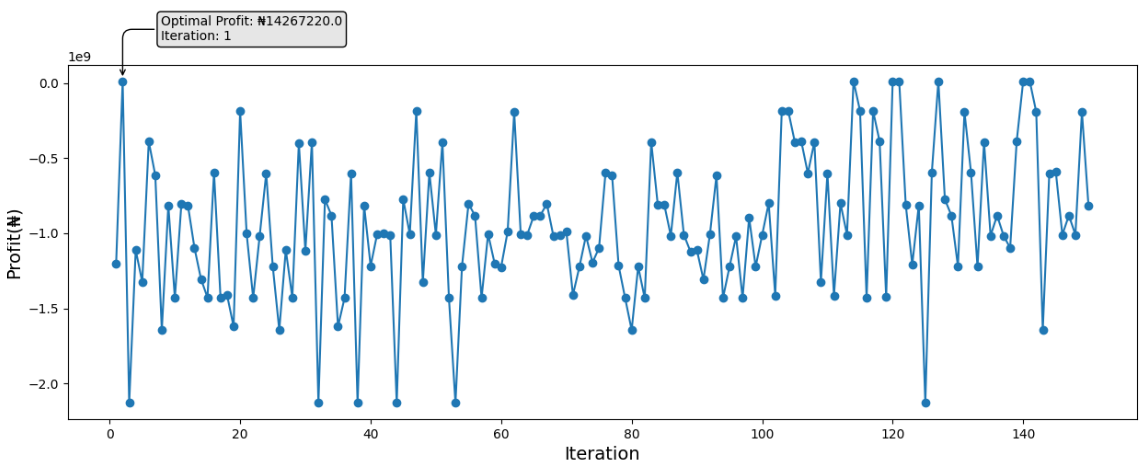

Figure 15. Plot of Profit against Iteration for the 120g noodles product

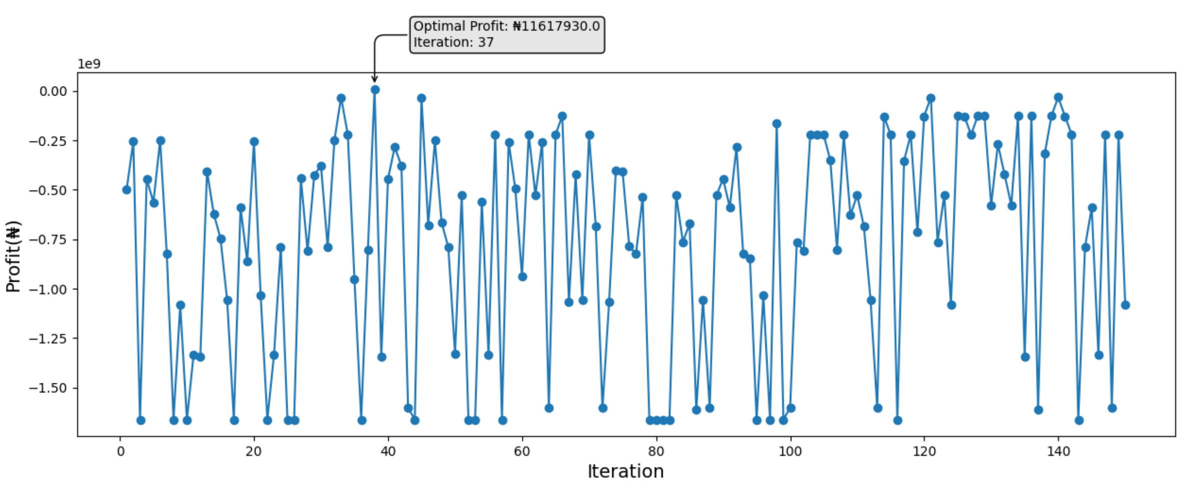

Figure 16. Plot of Profit against Iteration for the 180g noodles product

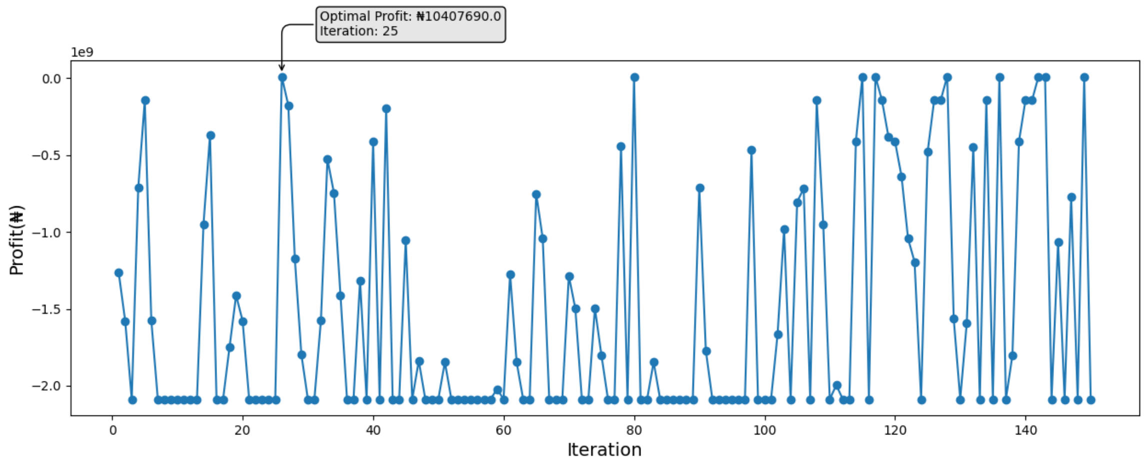

Figure 17. Plot of Profit against Iteration for the 280g noodles product

Routing optimization of the noodles manufacturing supply chain: The routing optimization model in Section 2.3 was applied in determining the most cost-effective routes for distributing products between the three distribution centers and five sub-distribution centers of the noodles manufacturing supply chain. The supply chain was optimized by considering only one product at a time. Figures 18-22 are bar charts that show, respectively, the quantity of 70g, 100g, 120g, 180g, and 280g noodles each distribution warehouse is expected to supply to each sub-distribution warehouse in order to achieve the minimal monthly total cost of distribution warehouse operation and cost of transportation of products to sub-distribution warehouses. The figures show that the data-driven routing optimization model can be used to analyze different inventory policies, routing strategies, and supply chain configurations to minimize costs. From the results, distribution warehouse 1 better caters to the demands of the noodles manufacturing supply chain because it has more capacity and is closer to the demand points. The optimization model determines warehouse allocations and routing decisions that jointly minimize the objective function by optimally balancing inventory, warehousing, and transportation costs. Rather than reducing each cost element independently, the model identifies the most cost-effective combination of allocation and routing strategies that minimizes overall supply chain cost while meeting all operational constraints. This integrated optimization approach enhances supply chain performance by reducing operating expenses, improving resource utilization, and achieving the minimum total cost.

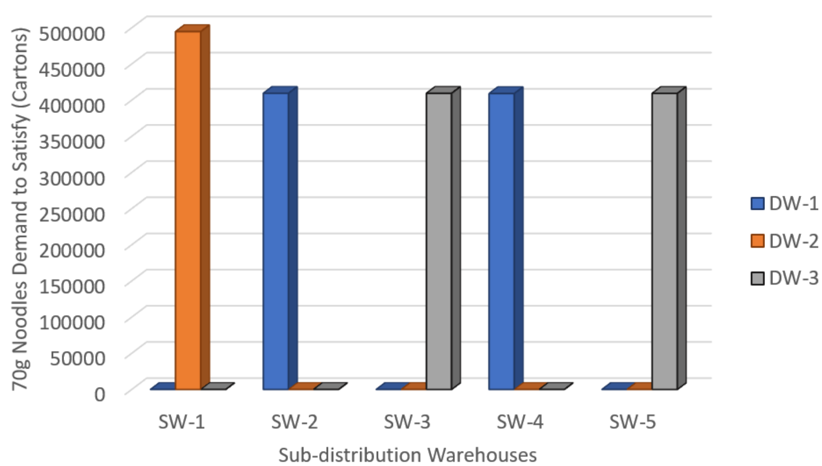

Figure 18. Quantity of 70g noodles each distribution warehouse (DW) is expected to supply to each sub-distribution warehouse (SW)

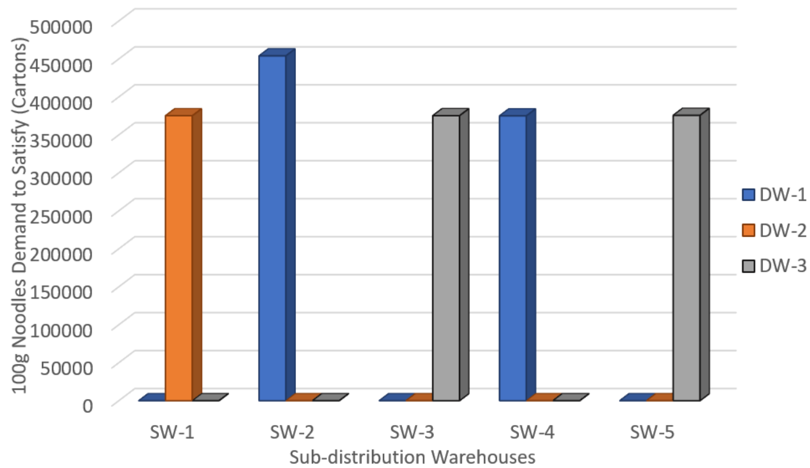

Figure 19. Quantity of 100g noodles each distribution warehouse (DW) is expected to supply to each sub-distribution warehouse (SW)

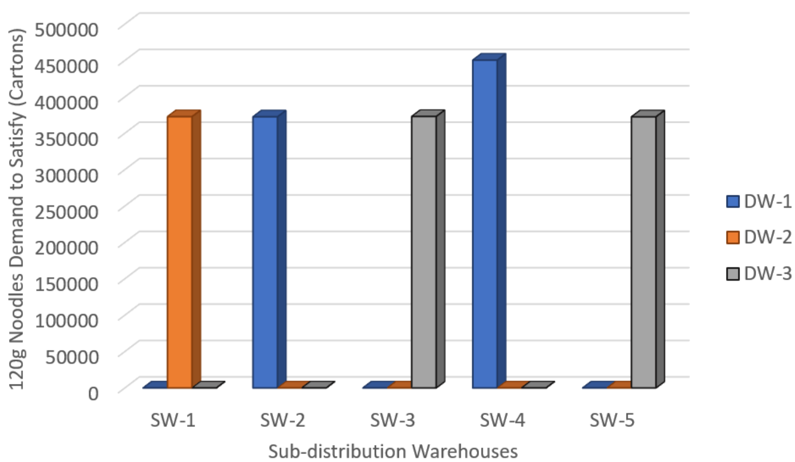

Figure 20. Quantity of 120g noodles each distribution warehouse (DW) is expected to supply to each sub-distribution warehouse (SW)

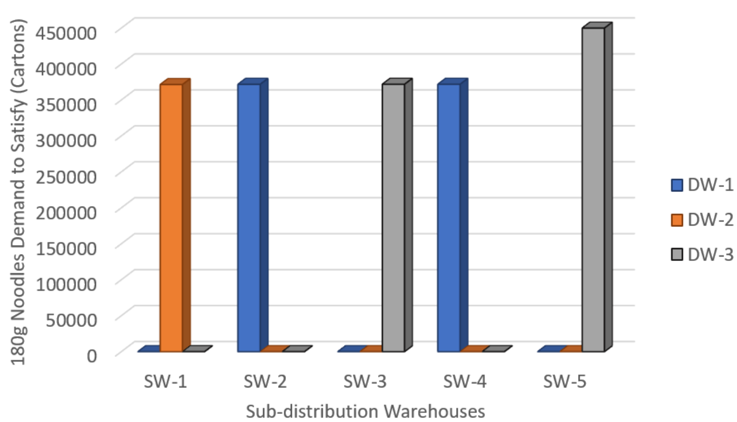

Figure 21. Quantity of 180g noodles each distribution warehouse (DW) is expected to supply to each sub-distribution warehouse (SW)

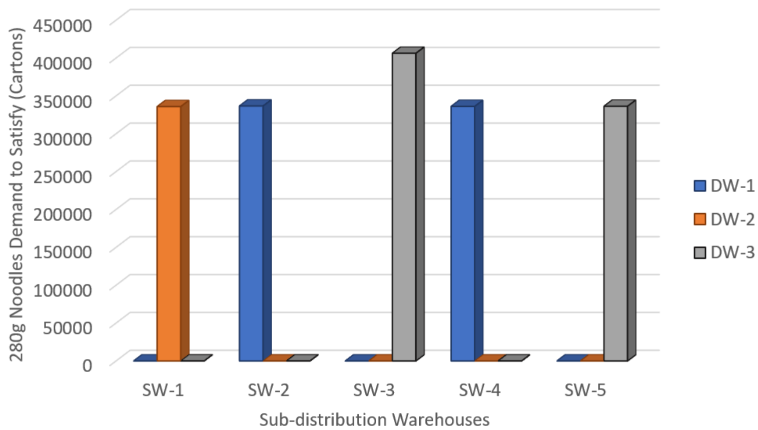

Figure 22. Quantity of 280g noodles each distribution warehouse (DW) is expected to supply to each sub-distribution warehouse (SW)

Discrete event simulation of the noodles manufacturing supply chain: The discrete event simulation model was applied to data from the noodles manufacturing supply chain to compare average backorders in the current state with those in the optimized state. The discrete event simulation model was also validated to ascertain its effectiveness in analyzing the noodles manufacturing supply chain using the equations in section 2.5. To verify the Arena model, various verification activities were conducted such as conducting an inspection and walk-through of the simulation program logic, inspection of code printouts, graphics and sample path routes while performing simulation test runs, to verify that the underlying program logic is accurate, as well as carrying out basic consistency checks, including sanity checks, as well as more sophisticated checks of theoretical relationships among statistics to be predicted. To validate the Arena model, ten (10) randomly selected observations of processing time for the 70g noodles product were compared with the processing time from ten (10) replications of the Arena model using a two-sample t-test in Minitab statistical software. Table 2 shows the observed and simulated processing time data for the 70g noodles product. The two-sample t-test at the 0.05 significance level yielded a p-value of 0.8767, which is greater than 0.05; therefore, the null hypothesis cannot be rejected, suggesting that the model is valid. The Arena model was run, and the current inventory policy of the noodles manufacturing supply chain was compared with the optimized inventory policy. Figure 23 shows a comparison of average backorders for the five (5) products handled by the supply chain under the current inventory policy and the optimized inventory policy. Backorders are orders that are not immediately satisfied due to insufficient inventory. From Figure 23, the optimized inventory policy reduced backorders for the 70g noodles, 100g noodles, 120g noodles, 180g noodles, and 280g noodles products by 3.5%, 12%, 20.6%, 7.1%, and 11.9%, respectively.

Table 2. Data of observed processing time and simulated processing time for the 70g noodles product

S/No.	Observed Processing Time (Seconds)	Simulated Processing Time (Seconds)
1	300	331.62
2	294	279.30
3	300	340.62
4	312	280.86
5	306	291.48
6	294	324.36
7	300	324.84
8	318	302.76
9	330	281.70
10	288	297.54

3.3 Sensitivity analyses

Sensitivity analysis was performed by determining the profit after varying one model parameter at a time by ±25% while keeping all other parameters constant. The sensitivity analysis revealed that the profit increases with higher selling prices and demand, and this profit is only limited by the model constraints. The analysis also revealed that higher production, ordering, warehousing and transportation costs tend to increase the total cost significantly, limited by only the model constraints. Figure 24 shows the sensitivity analyses plot considering selling price, demand, production cost, ordering cost, warehousing costs, and transportation cost.

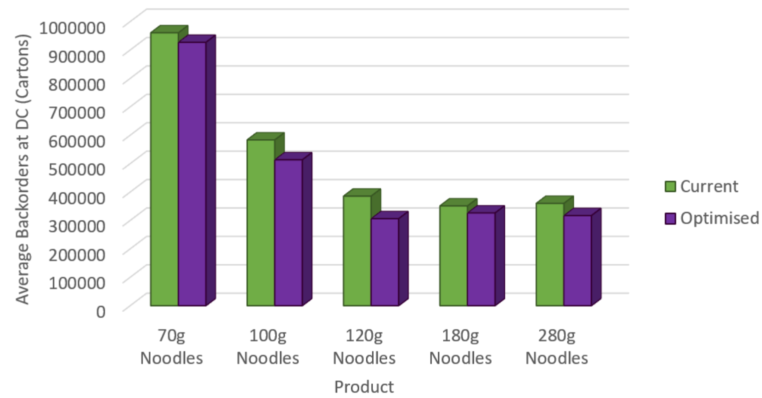

Figure 23. Comparison of the average backorders for the five (5) products of the noodles manufacturing supply chain considering the current inventory policy and the optimized inventory policy

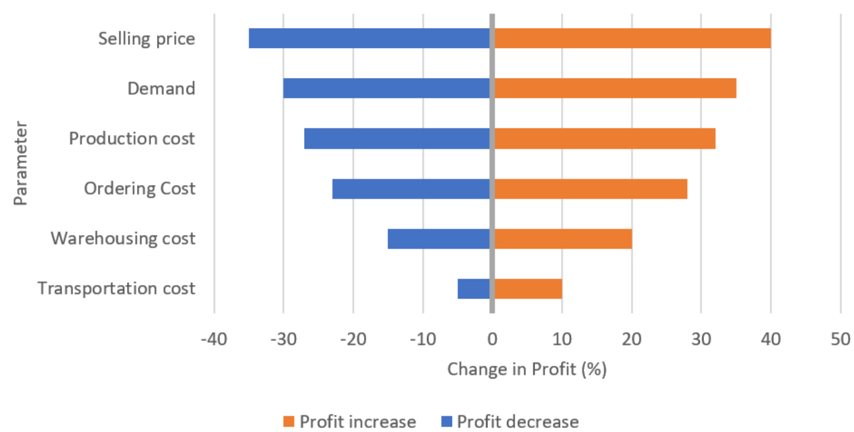

Figure 24. Sensitivity analyses

From Figure 24, the selling price has the largest impact on profitability: higher selling prices result in more profit, and lower selling prices result in less profit. This is because additional revenue from more sales offsets costs and improves the supply chain's bottom line. Moreover, higher demand significantly increases profit, and lower demand negatively affects profit. This is because revenue generally increases as more products are sold. Again, the production cost also considerably affects the profit. This is because production cost is a direct expense incurred in manufacturing, and an increase in production cost reduces the profit earned from each product sold. Ordering cost moderately affects profitability, with higher ordering costs reducing overall profit. However, the impact of ordering cost is smaller because ordering cost is incurred per order rather than per unit. Although an increase in warehousing cost affects overall profit, it does so to a lesser extent. This is because significant additional warehousing costs are incurred only when inventory levels far exceed initial warehousing provisions. Finally, the transportation cost is the least influential among the parameters. This is because products are distributed based on proximity to distribution and sub-distribution warehouses. Therefore, the framework selects the distribution warehouses that should supply specific sub-distribution warehouses based on the least transportation cost incurred. The framework is robust, as changes in input parameters lead to only minor variations in its objective functions and decision variables while all operational constraints remain satisfied.

3.4 Implications of the OKW optimization framework for decision-making

The findings of this study have significant managerial implications for manufacturing organizations seeking to improve supply chain performance through data-driven inventory and routing decisions. The proposed Optimizing Known Warehouses (OKW) framework provides supply chain managers with a systematic decision-support tool for simultaneously determining optimal inventory policies and vehicle routing strategies across multi-echelon, multi-product networks. By integrating Bayesian optimization with linear programming, managers can identify inventory policies that substantially reduce backorders while maintaining higher service levels without relying on extensive trial-and-error approaches. The observed reductions in backorders across both the water bottling and noodles manufacturing supply chains demonstrate that managers can improve product availability, minimize lost sales, and enhance customer satisfaction through scientifically derived inventory policies. Furthermore, the discrete event simulation component enables managers to evaluate proposed policy changes in a virtual environment before implementation, thereby reducing the financial risks associated with operational changes and supporting evidence-based strategic decision-making. The framework also enables more efficient allocation of organizational resources by helping managers balance inventory holding costs, transportation efficiency, and service performance across multiple products and warehouse locations. Since Bayesian optimization requires fewer function evaluations than many conventional metaheuristic algorithms, organizations can obtain high-quality inventory decisions with lower computational effort, making the approach suitable for periodic inventory policy reviews in dynamic manufacturing environments. The methodology additionally supports continuous improvement initiatives by allowing managers to evaluate alternative supply chain configurations under varying demand and operational conditions, thereby strengthening organizational resilience and responsiveness to market uncertainties.

From an industrial perspective, the proposed framework offers a practical optimization methodology that can be adopted across a wide range of Fast-Moving Consumer Goods (FMCG) manufacturing industries, including beverage, food processing, pharmaceuticals, consumer packaged goods, and other multi-product manufacturing sectors. The combination of mathematical optimization and discrete event simulation provides an integrated digital decision-support platform that aligns with Industry 4.0 initiatives by enabling data-driven planning, predictive analysis, and intelligent operational optimization. The framework's flexibility allows organizations to adapt it to different warehouse structures, transportation networks, and inventory management policies without fundamentally modifying the optimization procedure. The reduction in backorders achieved in both case studies indicates that widespread adoption of the framework could improve overall supply chain reliability, increase product availability, reduce emergency replenishments, and enhance distribution efficiency throughout manufacturing networks. Improved routing decisions can further reduce transportation costs, vehicle utilization inefficiencies, and delivery lead times while supporting more sustainable logistics operations through better resource utilization. Moreover, the simulation component provides industries with a low-risk environment to evaluate future investments, network expansions, warehouse additions, or policy modifications before physical implementation, reducing implementation risks and improving capital investment decisions.

Compared with recent hybrid optimization frameworks, the OKW framework tends to provide a more complete optimization strategy that considers multiple echelons and products in real-world manufacturing supply chains. For example, in the optimization framework by Rachman et al. [44], although reinforcement learning, similar to the Bayesian optimization algorithm, is used, multi-product distribution to downstream echelons such as sub-distribution centers is not considered. Therefore, their models cannot easily be applied to supply chains with more than one product without major adjustments. Although the framework by Masanta et al. [45] considered multiple products, their models focus on supply chains that distribute batch-produced vehicles and do not capture the variability experienced in extremely high-volume production and distribution settings such as the FMCG industry. The OKW framework addresses this by optimizing supply chains that handle FMCG products, which may be seasonal and have rapidly changing demand. In the framework by Vicente et al. [46], inventory planning was discussed; however, routing optimization, which is crucial in supply chain optimization, was not considered. In contrast, the OKW framework considers routing optimization for multi-echelon, multi-product supply chains extensively through the OKW routing optimization model. Routing optimization is crucial because it identifies the most efficient paths for vehicles, lowering operational costs and increasing on-time delivery within manufacturing supply chains. Overall, while the OKW framework is a scalable and transferable optimization methodology that bridges the gap between theoretical optimization models and industrial implementation, its main limitation is that, because of the mathematical formulation of its models, it can mostly be applied to FMCG manufacturing supply chains. Therefore, further research can develop or adapt the models in this study to optimize supply chains in areas beyond the FMCG sector, such as service, construction, waste, pharmaceutical/healthcare, petroleum, energy, and agricultural supply chains. By integrating Bayesian optimization, linear programming, and discrete event simulation into a unified framework, manufacturing organizations gain a robust decision-support system that improves operational efficiency, enhances customer service levels, strengthens supply chain resilience, and increases long-term competitiveness in increasingly complex and uncertain manufacturing environments.

4. Conclusion

This study developed and validated the Optimizing Known Warehouses (OKW) framework as an integrated optimization approach for analyzing and improving the performance of multi-echelon, multi-product manufacturing supply chains. The framework combines Bayesian optimization for inventory policy optimization, linear programming for vehicle routing optimization, and discrete event simulation for evaluating the operational impacts of alternative supply chain configurations. Applying the framework to two FMCG manufacturing supply chains demonstrated its effectiveness in reducing backorders and improving order fulfillment across multiple product categories. The results showed that the optimized inventory policies significantly reduced backorders for both the water bottling and noodles manufacturing supply chains, confirming the proposed framework's ability to enhance inventory management and customer service performance. By integrating mathematical optimization with simulation modeling, the framework enables organizations to determine optimal operational policies while assessing their practical implications before implementation. This integrated approach reduces the risks associated with operational changes and provides a comprehensive decision-support tool for supply chain planning and optimization. The study contributes to the existing body of knowledge by presenting a novel hybrid optimization framework that combines Bayesian optimization, linear programming, and discrete event simulation within a unified methodology for inventory and routing optimization. Unlike conventional approaches that optimize inventory or transportation independently, the proposed framework considers these decisions collectively, thereby providing a more realistic representation of manufacturing supply chain operations. Furthermore, the study demonstrates the applicability of Bayesian optimization as an efficient technique for determining optimal inventory policies in complex supply chain environments. From a practical perspective, the framework provides manufacturing organizations with a scalable and adaptable methodology that can be implemented across various Fast-Moving Consumer Goods (FMCG) industries and other multi-echelon supply chain networks. Its flexibility allows practitioners to evaluate different inventory policies, routing strategies, and supply chain configurations while supporting informed managerial decision-making to improve operational efficiency, reduce shortages, and enhance customer satisfaction. This study is significant because it provides a procedure for inventory and routing optimization using the Bayesian optimization algorithm and linear programming, respectively, as well as a Discrete Event Simulation model for analyzing inventory policies. Although the proposed framework was validated using two manufacturing supply chains, future research could extend its application to larger and more complex supply chain networks involving multiple manufacturers, suppliers, distribution centers, and retailers under stochastic demand conditions. Future studies may also integrate additional optimization objectives, such as minimizing carbon emissions, improving supply chain resilience, reducing transportation costs, or incorporating real-time data from Internet of Things (IoT) devices and digital twin technologies. Also, based on the foregoing research, the following areas of further research are recommended: the discrete event simulation model developed in this study cannot be easily applied to optimizing closed-loop supply chains. Therefore, further studies can develop models that can be applied to optimize closed-loop supply chains. The models in this study need to be developed or adapted to optimize supply chains beyond the Fast-Moving Consumer Goods (FMCG) sector, such as service, construction, waste, pharmaceutical/healthcare, petroleum, energy, and agricultural supply chains. Further research can determine how other machine learning optimization methods, such as reinforcement learning, can be used for inventory optimization. Other machine learning optimization methods can also be compared with Bayesian optimization and linear programming to determine the most effective method or combination of methods for reducing backorders, eliminating waste, decreasing costs, and increasing overall profitability across supply chain categories.

Ethical issue

The authors are aware of and comply with best practices in publication ethics, specifically regarding authorship (avoidance of guest authorship), dual submission, figure manipulation, competing interests, and compliance with research ethics policies. The authors adhere to publication requirements that the submitted work is original and has not been published elsewhere in any language.

Data availability statement

The manuscript contains all the data. However, more data will be available upon request from the corresponding author.

Conflict of interest

The authors declare no potential conflict of interest.

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