Future Sustainability
Open Access Journal
ISSN 2995-0473

February 2026| Volume 04 | Issue 01 | Pages 45-55
Journal homepage: https://fupubco.com/fusus
https://doi.org/10.55670/fpll.fusus.4.1.5

Future Publishing LLC

Article

A low-order dynamical model for delayed thermal drawdown in subsurface energy systems

Coskun Firat*

Istanbul Technical University, Energy Institute, Türkiye

ARTICLE INFO

Article history:
Received 09 November 2025
Received in revised form
23 December 2025
Accepted 24 January 2026

Keywords:
Geothermal reservoirs, Thermal drawdown, Low-order dynamical modeling, Time-delay systems, Reservoir sustainability

*Corresponding author
Email address:
coskun.firat@itu.edu.tr

DOI: 10.55670/fpll.fusus.4.1.5

ABSTRACT

Long-term thermal drawdown is a fundamental constraint on the sustainability of subsurface thermal energy systems, yet its onset often does not become apparent for several decades after exploitation begins due to large thermal inertia and slow heat transport processes. This delayed response complicates sustainability assessment and may lead to overestimation of system longevity when early operational data are used. Although high-fidelity numerical simulators can capture delayed thermal behavior, their computational cost and limited interpretability restrict their usefulness for rapid, conceptual analysis at the system level. This study presents a simplified low-order dynamical model to examine delayed thermal drawdown in subsurface energy systems, with geothermal reservoirs considered as a primary application. The system is represented by a lumped thermal state driven by heat extraction and gradual geothermal recharge, with an explicit time-delay term introduced to account for geological memory and delayed thermal response. Analytical and numerical investigations using synthetic production scenarios show that significant thermal drawdown emerges only when production history changes, explaining prolonged early-stage stability followed by later temperature decline. The proposed framework is intended as a screening-level and educational tool that complements high-fidelity numerical simulations and supports long-term management of thermal energy systems.

1. Introduction

Geothermal energy is often considered a reliable, low-carbon source of electricity that can deliver constant baseload power from a very small surface footprint. Unlike other variable renewables, geothermal systems are insensitive to weather patterns and therefore especially well-suited to planning horizons and long-term decarbonization goals [1]. Its power generation position, however, depends on the thermal evolution of subsurface reservoirs over years to decades, wherein gradual heat depletion may impact the performance of the long-term production. The thermal energy extracted from a geothermal reservoir is obtained essentially from the heat stored in the coupled rock-fluid system with additional contributions from conductive and advective recharge derived from the formations surrounding the reservoir [2]. If long-term heat extraction is greater than natural recharging, reservoir temperatures will gradually decrease over time, with a consequent reduction of production enthalpy and, therefore, power output. Maintaining the temperature of the reservoir is hence crucial for the long-term technical and economic feasibility of geothermal operations [3].

A central challenge in managing geothermal reservoirs is the fact that thermal depletion does not immediately occur with the commencement of production. Instead, temperature decline, often referred to as thermal drawdown, sometimes shows up after a significant delay-a number of decades after initial exploitation [4]. This retardation effect is due to the great thermal inertia of the reservoir, sluggish heat flow through the porous or fractured rock, and the gradual interaction between the exploitation zone and its surrounding heat sources. Therefore, thermal drawdown represents a time-delayed, system-level response to cumulative extraction history and slow transport processes in the subsurface and should not be considered in terms of short-term operational effects. Properly capturing this delay behavior is integral to making credible sustainability assessments, planning long-term production, and mitigating risks in geothermal developments [5]. Historically, thermal drawdown in geothermal reservoirs has been investigated by high-fidelity numerical models coupling heat transport and fluid flow in two or three spatial dimensions. Thermo-hydraulic (TH) and thermo-hydro-mechanical (THM) simulations have been very useful for detailed site-specific analyses, reservoir design, and regulatory assessment [6].

Such models can also accommodate complex geological heterogeneities and highly detailed well configurations with high fidelity. However, high-fidelity numerical models face practical limitations in being used to assess long-term sustainability. Mainly, they are computationally intensive for simulating multi-decade reservoir operation and/or performing extensive parametric and uncertainty analyses [7]. This makes them impractical in early-stage feasibility studies as well as rapid scenario screening. While such models can simulate delayed thermal drawdown, the root causes of long-time delays are implicitly buried within numerical solutions, often providing limited physical intuition about the dominant system-level controls [8]. Finally, the sophistication of high-fidelity models inhibits the systematic investigation of alternative production strategies, reinjection schemes, and long-term operational scenarios; hence, such studies are rather time-consuming [9]. Hence, there is yet a gap between what is possible using detailed numerical simulations and the need for simple, interpretable models that nonetheless capture the essence of geothermal reservoir long-term thermal dynamics. In short, "while high-fidelity numerical models are indispensable, they are not well-suited for rapid conceptual analysis or long-term sustainability screening."

The objective of this study is to develop a simplified yet physically interpretable framework for analyzing long-term thermal drawdown in geothermal reservoirs. Rather than replacing detailed numerical simulations, the proposed approach is designed to complement existing modeling tools by offering a low-order representation of reservoir-scale thermal dynamics that explicitly captures the delayed response of the system. Specifically, this work makes the following contributions:

- A low-order dynamical model is proposed to describe geothermal reservoir thermal drawdown using a small number of physically meaningful state variables.
- The geothermal reservoir is interpreted as a slow thermal state variable with large thermal inertia, driven by production and reinjection as external forcing terms.
- Time-delayed thermal response is explicitly incorporated into the model formulation, allowing direct representation of the delayed onset of thermal drawdown observed in geothermal systems.
- The qualitative behavior of the proposed model is shown to be consistent with known geothermal field behavior and established physical understanding of subsurface heat transfer.

1.1 Reduced order and conceptual modeling approaches

A range of reduced-order and conceptual models have previously been employed to study geothermal reservoir behavior, including lumped-parameter reservoir models, thermal tank representations, and decline-curve-type approaches. More recently, reduced-order models have been developed as surrogate approximations trained on high-fidelity thermo-hydraulic simulations, enabling rapid evaluation of operational scenarios. Unlike surrogate reduced-order models that rely on numerical training data, the approach presented is physics-based and explicitly embeds delayed thermal dynamics due to geological memory and slow heat transport. This distinction allows for delayed drawdown behavior to be represented transparently and interpreted directly in terms of physical mechanisms rather than learned correlations. Conceptual clarity and physical insight rather than site-specific predictive accuracy is the focus of this study. The framework proposed is appropriate for rapid conceptual screening and educational analysis of thermal systems with long characteristic time scales, and may be extended in future work toward field calibration, uncertainty quantification, and coupling with more detailed models.

2. Conceptual framework

2.1 Geothermal reservoir as a dynamical system

Geothermal reservoirs are complex subsurface systems where thermal, hydraulic, mechanical, and chemical processes interact in a coupled way. The behavior at scales relevant for sustained energy production is often dominated by the balance between extracted heat and slow thermal recharge from the surrounding geological environment [10]. This decoupling of the microscopic complexity from the macroscopic behavior is what justifies the use of reduced order representations emphasizing effective system level variables. In this study, the geothermal reservoir is conceptualized as a dynamical system whose long-term evolution can be described using a small set of physically meaningful variables. Rather than resolving spatial temperature distributions explicitly, the reservoir is represented by an effective or bulk thermal state variable that captures the dominant thermal energy available for production. Under this abstraction, the key physical components of the geothermal system can be mapped to elements of a low-order dynamical model as summarized in Table 1.

Table 1. Mapping of physical geothermal system components to low-order dynamical model elements

Physical Meaning	Model Component
Reservoir rock and fluid	Thermal state variable
Fluid extraction and injection	External forcing term
Conductive heat inflow / geological recharge	Slow recovery mechanism

The reservoir thermal state is represented by an effective reservoir temperature, denoted as $T_{res}(t)$, which reflects the average thermal energy content of the actively exploited reservoir volume. This temperature does not correspond to a single point measurement, but rather to a lumped parameter representing the collective thermal response of the coupled rock–fluid system [11]. The production process is represented by the mass or volumetric extraction rate $Q(t)$ which is the external forcing that removes the thermal energy from this system. $Q(t)$ can change quickly as a function of operational decisions or market conditions or plant controls strategies [12]. This thermal recharge takes place mainly through conductive heat transfer from the surrounding formations and, in special cases, by advective inflow from deeper or lateral parts [13]. At the scale of the reservoir, these processes operate over long periods of time and can be represented by a slow process of regeneration opposing the thermal drawdown. The conceptual view of the geothermal reservoir as a forced dynamical system, with one dominant thermal state variable, allows for an analytical isolation of fundamental physics controlling long-term thermal drawdown without resolving detailed spatial temperature fields. On time scales relevant to sustained geothermal energy production, the reservoir can be considered a dynamical system whose main behavior is controlled by the balance between extracted heat and slow thermal recharging. The system can then be described, instead of resolving its spatial temperature distributions, by means of such an effective thermal state variable, forced by external conditions and delayed subsurface processes. Figure 1 shows a proposed conceptual framework wherein a geothermal reservoir is modeled as a delayed dynamical system that explicitly incorporates thermal inertia and geological memory.

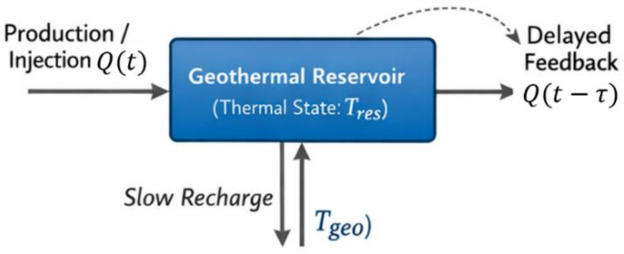

Figure 1. Conceptual representation of a geothermal reservoir as a delayed dynamic system

We define an effective thermal state variable T_{res} that describes the reservoir and is forced by fluid production and injection $Q(t)$. Heat extraction is an external forcing, while natural thermal recharges over very long timescales from the surrounding geological formation, represented by T_{geo}, dampens long-term depletion. The delayed feedback $Q(t-\tau)$ models the time-lagged thermal response to past extraction due to slow heat transport and thermal front propagation and rock–fluid heat exchange processes. This delay represents geological memory rather than any operational control. In this context, the rates of production and injection are external driving variables that could change considerably in a very short operational time period, whereas the thermal state of the reservoir would respond very slowly because of significant thermal inertia and heat transport mechanisms taking place much later. It is this separation between fast forcing and slow response of the system that forms one of the distinctive features of geothermal reservoirs and also a well-founded reason for their low-order dynamical modeling, as described in greater detail later.

2.2 Time-scale separation

A characteristic of geothermal reservoir dynamics is a strong separation between operational and thermal time scales. Production and injection rates change either as a result of day to month timescale operational decisions, while the thermal state of the reservoir itself changes over years to decades due to slow heat transport and significant thermal inertia [14,15]. This timescale separation is the basis for delayed thermal drawdown and provides a key rationale for the application of low-order dynamical models. Figure 2 shows how production and injection rates $Q(t)$ might change rapidly over operational time scales of days to months (top), while the effective reservoir temperature $T_{res}(t)$ responds slowly over thermal time scales of years to decades (bottom).

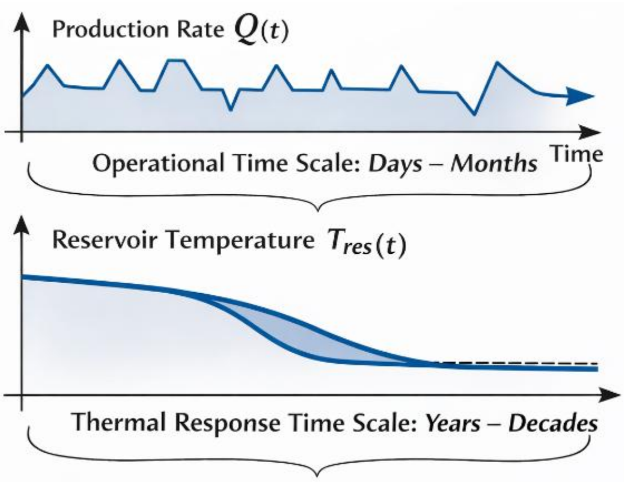

Figure 2. Schematic illustration of time-scale separation in geothermal reservoir systems

This large difference in time scales causes a delayed thermal drawdown and supports seeking low-order dynamic representations. The implication of this is that the thermal state of the reservoir has high inertia, responding only gradually to changes in extraction or injection conditions. This is one root cause of why thermal drawdown often develops well after the start of geothermal production [16]. From a modeling perspective, timescale separation offers a rigorous basis for low-order representations. When fast processes primarily operate as external drivers and slow processes dominate state evolution, system dynamics can often be captured using a limited set of state variables governed by ordinary differential equations [17]. In geothermal reservoirs, the effective reservoir temperature evolves slowly under the cumulative influence of production history rather than instantaneous operating conditions.

Importantly, the delayed onset of thermal drawdown is not anomalous but represents a natural consequence of this time-scale separation. Early-stage production data may indicate stable temperatures even as thermal depletion transpires invisibly within the reservoir. Low-order models that explicitly acknowledge slow thermal dynamics are thus well-suited for analyzing long-term sustainability and delayed risk [18]. Time-scale separation is used in the present framework to isolate the dominant long-term behavior of geothermal reservoirs at the expense of intentionally neglecting short-term fluctuations that would have a minor impact on the overall trend of thermal depletion. The method allows for transparent analysis of delayed thermal drawdown and serves as the basis for the dynamical model described in the subsequent sections.

3. Mathematical model formulation

3.1 Governing equation

Following the conceptual framework developed in Sec. 2, long-term thermal evolution of a geothermal reservoir is represented through a low-order dynamical equation for effective reservoir temperature. The objective is not an accurate reproduction of detailed spatial temperature fields but the representation of the dominant system-level response of a reservoir to sustained heat extraction and gradual thermal recharge.

The governing equation is expressed in the following general form:

$$\frac{dT_{res}}{dt} = -\alpha Q(t) + \beta\left(T_{geo} - T_{res}\right) - \gamma Q(t-\tau) \tag{1}$$

where T_{res} denotes the effective reservoir temperature, $Q(t)$ is the fluid extraction rate, and T_{geo} represents the background geothermal temperature of the surrounding formation. Each term in this equation corresponds to some physically interpretable process:

- $-\alpha Q(t)$ accounts for the instantaneous loss of thermal energy due to fluid production. The parameter α lumps fluid properties, heat capacity, and the effective heat exchange between the produced fluid and the reservoir rock [19], representing the instantaneous cooling effect produced by extraction.
- $\beta\left(T_{geo} - T_{res}\right)$ accounts for the slow thermal recharging of the reservoir due to conductive and large-scale advective heat transfer from surrounding geological formations. The parameter β quantifies the strength of this recovery mechanism and reflects the thermal coupling between the reservoir and its environment [20].
- The term $-\gamma Q(t-\tau)$ introduces a time-delayed cooling effect, capturing the fact that thermal depletion due to production is not instantaneous at the reservoir scale. The coefficient γ denotes the magnitude of the delayed thermal impact, while the delay time τ captures the characteristic lag between production activity and an observable temperature decline [21].

In particular, the governing equation is intentionally written in a general form. The parameters α, β, γ, and τ are properties of no specific geothermal field, or lithology, or well configuration; rather, they are effective properties that describe the integrated thermal behavior of the system as a whole.

3.2 Interpretation of the time delay

Explicit incorporation of a time-delay term is a defining aspect of the proposed formulation that reflects fundamental physical processes governing the behavior of geothermal reservoirs. In geothermal systems, thermal signals generated by production propagate through the subsurface over long time frames to yield delayed temperature responses at the reservoir scale. One of the contributing mechanisms is thermal front propagation. Cooling of the formation associated with cooler reinjected fluids or thermal depletion zones will propagate through the rock of limited thermal diffusivity and along tortuous pathways of flow very slowly [22]. As a result, cooling associated with a particular production rate may not be observed until years or even tens of years later. A second mechanism is the lag in heat exchange between the circulating fluid and the surrounding rock matrix. Even for rapidly changing fluid temperatures, the bulk thermal state of the reservoir develops quite slowly due to the high heat capacity of the coupled rock-fluid system and imperfect thermal equilibration [23]. Further delaying the observable impact of extraction on reservoir temperature. In combination, these processes give rise to a kind of geological memory of the reservoir's thermal state. The present operating conditions cannot solely determine the current thermal state but also depend on the integrated history of all previous production and injection [24]. The time-delay term $Q(t-\tau)$ represents this memory effect concisely, making previous extraction rates influential in temperature development. By explicitly incorporating delay, the model captures a key observation arising from geothermal fields: most early production periods have near-flat temperatures while thermal depletion processes advance and appear as rising temperatures only later in the operational life [25]. These types of delayed responses are essential to the understanding of long-term sustainability and risk in geothermal energy production.

3.3 Model assumptions and scope

The present model represents a low-order, lumped-parameter representation of the thermal behavior of geothermal reservoirs. In the light of this characterization, several simplifying assumptions are recognized and accepted. First, the reservoir is treated as a lumped thermal system, described by a single effective temperature, T_{res}. Spatial heterogeneity in temperature, permeability, and fracture architecture is not resolved explicitly; instead, such heterogeneity is implicitly captured through effective model parameters [26, 27]. Homogeneous effective properties are assumed; parameters like α, β, and γ stand for averaged thermal and hydraulic characteristics of the actively exploited reservoir volume. Those may be different for various geothermal systems but, for the purpose of conceptual analysis, it is presumed that they are constant in time.

Third, the model is designed to provide qualitative and interpretive results rather than sharp quantitative predictions. Its central aim is to provide insight into the mechanisms of delayed thermal drawdown (DTD) and to perform a rapid pre-screening of sustainability while carrying out conceptual production strategy evaluation. The detailed forecasting and site-specific optimization tasks are left to high-fidelity numerical simulations [28]. Within this framework, the model provides physical interpretability and insight into the long-term behavior of geothermal reservoirs and serves as a basis for further developments including calibration, uncertainty analysis, and coupling to more detailed models.

4. Analytical behavior and regime analysis

4.1 Steady-state behavior

Insight into the long-term behavior of the proposed system can be obtained by examining its steady-state solution. For constant extraction rate $Q(t) = Q_0$, the equilibrium reservoir temperature T_{res}^* is defined by:

$$\frac{dT_{res}}{dt} = 0 \tag{2}$$

Substituting into the governing equation yields

$$T_{res}^* = T_{geo} - \frac{(\alpha+\gamma)}{\beta} Q_0 \tag{3}$$

This formulation reflects a straightforward and intuitive relationship between the equilibrium temperature of the reservoir and the rate of drawdown. Steady-state temperatures are lower for long periods of increased production, while stronger thermal coupling to the surrounding geology-i.e., a larger β-reduces long-term drawdown. Importantly, the equilibrium temperature embodies the balance of cumulative heat extracted and gradual geothermal recharge rather than the short-term operational fluctuations. This supports the viewpoint above that the sustainability of a reservoir is a property of the whole system, which is further governed by the long-term averages rather than instantaneous production behavior. The steady-state solution also provides a convenient conceptual benchmark. While real geothermal reservoirs cannot attain true equilibrium in operational time frames, the equilibrium temperature defines a lower bound toward which the system tends under sustained extraction [29, 30].

4.2 Transient response

Though the steady state provides valuable information, the most important dynamics in geothermal reservoirs occur in the transient process towards the equilibrium state. Notably, the inclusion of the time delay completely changes the dynamical behavior of the model. After the start of production, the instantaneous cooling effect $-\alpha Q(t)$ affects the reservoir temperature, whereas the delayed effect $-\gamma Q(t-\tau)$ does not come into play until after a time delay of τ. During this period, recharge and reservoir capacity effects might be more important, simulating a "phase of stability or slowly declining temperatures" [31]. After the initiation of the delayed cooling effect, a "rapid decrease of temperatures might be observed" despite unchanged rates of production [32]. Depending upon the values of the parameters, several kinds of transient processes can occur:

- Monotonic Decay, where the temperature of the reservoir leaks monotonically to equilibrium.
- Delayed acceleration, representing gradual response rates until the onset of rapid decline rates after the effects of delay dominates.
- Overshoot, where the system is kept in balance by the replenishment of heat, but the delay in cooling causes the system to fall below the long-term path.
- Overshoot-like behavior, such regimes arise based on the combination of thermal inertia, recharge rate, and magnitude of delay, rather than changes in operational parameters.

The model therefore emphasizes that delayed thermal drawdown is a characteristic of the reservoir system itself rather than an operational issue [33].

4.3 Dimensionless interpretation

To improve clarity in terms of the underlying dynamics and to allow easier comparison of different geothermal resource types, the model can also be cast in dimensionless form. This reveals a compact subset of key parameters that control the behavior of the reservoir independent of any particular scales. Based on characteristic temperature difference ΔT, an extraction rate Q_0, and a time-scale $t_c = 1/\beta$, the dimensionless form of the underlying model equation consists of three key parameters. The thermal inertia number is given by:

$$\Pi_{TI} = \frac{1}{\beta t_c} \tag{4}$$

It represents the relative importance of reservoir thermal inertia compared to recovery processes. Large values of Π_{TI} correspond to systems with slow thermal response and strong memory effects [34]. The extraction intensity ratio:

$$\Pi_{EI} = \frac{(\alpha+\gamma)Q_0}{\beta \Delta T} \tag{5}$$

It quantifies the strength of thermal forcing relative to geothermal recharge. This parameter governs the magnitude of long-term drawdown and determines whether equilibrium temperatures remain economically viable [35]. The delay ratio:

$$\Pi_D = \frac{\tau}{t_c} \tag{6}$$

It measures the relative importance of delayed effects compared to the intrinsic recovery time of the reservoir. High delay ratio values indicate conditions under which drawdown might be hidden for long periods of time before being detectable [36]. The sets of dimensions listed above define typical zones in the behavior of geothermal reservoirs and form a useful basis for discussion of sustainability issues.

5. Illustrative numerical experiments

Numerical experiments using synthetic production scenarios were performed to test the behavior of the proposed low-order model. The aim is to characterize typical thermal responses and explain the part of delayed dynamics under controlled conditions, rather than to represent specific geothermal fields. The simulations use nondimensionalized parameters commonly adopted for geothermal systems in order to preserve generality and avoid site-specific assumptions. These numerical experiments are qualitative illustrations rather than quantitative predictions. Regular time-marching schemes with small steps are used in order to ensure smooth and stable solutions. No formal convergence analysis was pursued because numerical accuracy is not the focus of the present conceptual investigation.

5.1 Synthetic production scenarios

Three idealized production scenarios were considered to evaluate the impact of different operation profiles on the thermal history of the reservoir.

Constant production: In the first case, the extraction rate is held constant, $Q(t) = Q_0$; this would represent continuous operation. This setup provides a means of dealing with how thermal levels drop and settle over time.

Step increase in production: The second scenario introduces a step increase in extraction rate:

$$Q(t) = \begin{cases} Q_0, & t < t_s, \\ Q_1, & t \geq t_s, \end{cases} \tag{7}$$

where $Q_1 > Q_0$. This configuration mimics capacity expansion or operational intensification and allows examination of how delayed thermal effects respond to abrupt changes in production strategy [37].

Pulsed extraction: The third case considers pulsed or cyclical extraction, based on oscillations of the production rate that occur periodically. These oscillations can be developed from specific operational constraints, maintenance activities, and fluctuations in operational demand; therefore, they reveal interactions between particular operational variability and the long-term effects of associated thermal inertia [38]. The three cases together represent the spectrum of possible valid operational conditions and enable immediate interpretation of model processes.

5.2 Thermal drawdown profiles

First, we examine the long-term response for the reservoir in the case of constant production. This is a useful background for evaluating the lagged phenomenon. Although this case does not involve a lagged phenomenon, it illustrates the effects of continued extraction and the approach toward the long-term equilibrium. Reservoir temperature evolution during constant production is shown in Figure 3.

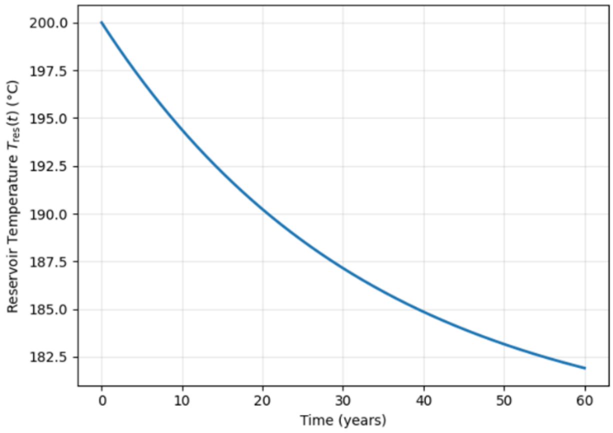

Figure 3. Reservoir temperature evolution under constant extraction rate. The temperature decreases gradually as cumulative heat extraction exceeds geothermal recharge, approaching a long-term equilibrium determined by the balance between extraction and recovery. Under constant forcing, delayed effects do not influence transient behavior, providing a baseline reference for subsequent analyses.

As indicated in Figure 3, continued extraction causes a progressive thermal drawdown due to the balance between heat removal and the slow geothermal recharge. For a steady extraction rate, the delayed term gives a constant offset with no change in the shape of the transient response. It follows that one important restriction of constant-production models is that delayed thermal effects manifest themselves only as the production history changes. We, therefore, introduce a step increase in extraction rate as a means of introducing a well-defined variation in the forcing history for elucidation and demonstration of geological memory; Figure 4 displays the sensitivity of the reservoir temperature evolution to the delay parameter. As shown in Figure 4, the reservoir temperature response remains indistinguishable across all delay values prior to the step change, reflecting the identical production history experienced by the system. After the step-in extraction rate, delayed cooling effects activate at different times depending on τ, leading to progressively later and more gradual drawdown for larger delay values. This behavior demonstrates that delayed thermal drawdown is governed by the interaction between production history and reservoir memory, rather than by instantaneous operating conditions. To clarify how geological memory works, we compared thermal responses of systems with and without delay when conditions were the same, because Figure 4 shows that the timing of thermal drawdown depends on the delay parameter, but it doesn't isolate the effect of delay itself.

As shown in Figure 5, it can be seen that the presence of a delay changes significantly the interpretation of early production rates. While the delayed system indicates a degree of thermal equilibrium in response to the operational shift, this is a transient effect which corresponds more to the onset of subsurface thermal evolution than a true equilibrium. When the delayed effect of the temperature is in effect, it is evident that the cooling rate is accelerated despite a lack of changes in operating conditions. The numerical experiments tend to imply sensitivity to parameters since changes in the delay term and the strength of recharge result in different qualitative patterns of thermal drawdown. A rigorous treatment of uncertainty quantification is outside the current work's scope. It is, however, relevant to future studies.

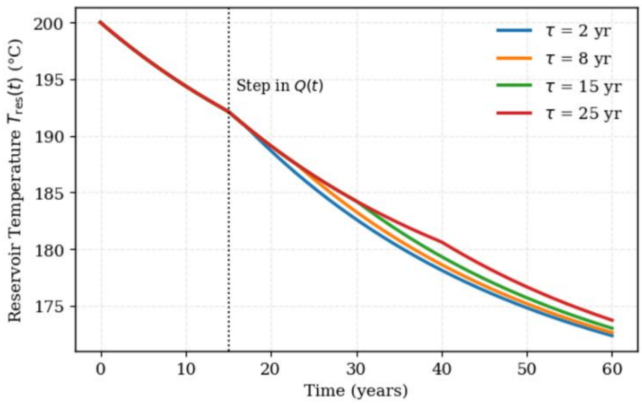

Figure 4. Sensitivity of reservoir thermal drawdown to the delay parameter τ under a step increase in extraction rate. All cases exhibit identical behavior prior to the operational change due to identical forcing history. Following the step in $Q(t)$, the onset and rate of temperature decline depend strongly on τ, demonstrating the role of geological memory in delayed thermal response.

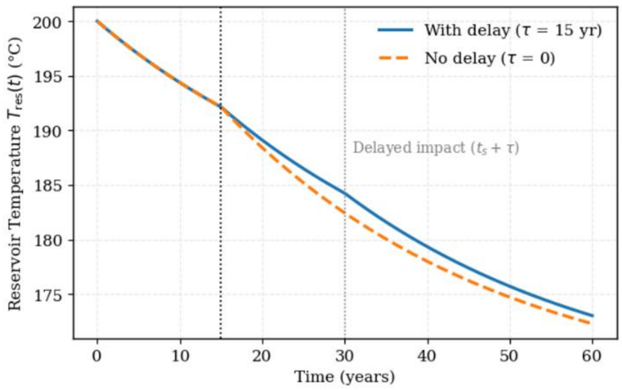

Figure 5. Comparison of immediate and delayed thermal response to a step increase in extraction rate. In the absence of delay (τ=0), the reservoir temperature responds immediately to the operational change. When delay is present, the thermal impact of increased extraction is postponed until approximately $t_s+\tau$, resulting in an apparent period of stability followed by delayed thermal drawdown.

5.3 Comparison with qualitative field behavior

The model captures typical patterns of thermal drawdown that can be found in well-documented geothermal resources, even in the absence of site-specific calibration. To be more precise, it captures a long-term initial phase of stagnation followed by a slow drawdown of temperatures for tens of years. Such patterns can be found in the so-called 'mature' geothermal resources such as 'The Geysers' in the USA, 'Icelandic high enthalpy systems,' or 'New Zealand geothermal reservoirs,' where thermal depletion only became manifest after very long-term exploitation [39-41]. The proposed model incorporates several qualitative characteristics described in the literature, which include:

- delayed onset of thermal drawdown,
- Sensitivity to sustained extraction rates,
- Short-term effects of operational variability on long-term trends of temperature. In particular, it is important to note that it is the character of the decay in temperature rather than the values that are highlighted by this model.

This agreement between model predictions and observation in the field lends physical credence to the low-order model without making use of databases that could be considered to be proprietary in nature [42].

6. Discussion

6.1 Physical insights

Analyses by both analytical and numerical scales provide a number of important physical insights into the long-term thermal behavior. In particular, strong thermal inertia in the reservoir at early times of production and inherently slow heat-transport processes preclude any immediate decline of temperature. Thermal recharge by surrounding formations and incomplete propagation of thermal fronts temporarily balance heat extraction, creating an apparent period of sustainability [43]. During this regime, operational indicators using short-term temperature measurements can strongly underestimate the risk of long-term depletion. Although this effect is negligible at early times, it builds up as time progresses; if delayed cooling mechanisms dominate, the system will eventually enter a thermal drawdown regime even under constant production rates. This transition might be linked not to operational changes but rather to time delays of the subsurface thermal processes controlled by geological memory and sluggish heat exchange [44]. From an operational viewpoint, this analysis reveals a profound risk: geothermal reservoirs may be driven far past a sustainability threshold before there is any significant wellhead temperature decline. It follows that actions taken during the first few production years can have irreversible long-term impacts, and that any planning and managerial decisions should carefully consider delayed system responses [45].

6.2 Implications for reservoir management

The low-order model developed in this work provides a number of implications for geothermal reservoir simulation in the context of decision-making under the lens of sustainability. Firstly, the modeled delayed response provides insight into the contribution of reinjection schemes. Delayed response in temperature could have limited effects in the short term regarding the impact of reinjection schemes on the temperature in the geothermal reservoir. Yet, such schemes could have significant effects in the long term through delayed response models [46]. Secondly, the implications of the steady state and dimensionless results emphasize the need to find sustainable rates of extraction that match the removal of heat with the natural replenishment of geothermal fluids. Rather than pursuing schemes that focus on the short-term maximization of production rates, producers should find schemes within which the rate of delayed drawdown is not extreme, and the equilibrium temperatures are economically feasible [47]. Thirdly, the framework suggests there could be early warning signals using system-level measures instead of real-time temperatures. For instance, a cumulative exhaustive history of system data, an estimate of system delays, or a change in the efficiency of gas recovery could potentially yield an earlier warning of impending drawdown than real-time temperatures alone could do [48]. While the model developed here is not designed for direct operational control, it does have a clear conceptual foundation that serves to clarify risks over the long term and inform the development of more detailed models.

6.3 Positioning relative to high-fidelity models

It is emphasized that the proposed low-order model does not strive to replace high-fidelity numerical simulators. Only fully developed thermo-hydraulic and thermo-hydro-mechanical models can serve site-specific design, regulatory compliance, and quantitative forecasting [49]. Rather, its strength lies in its complementary function: it allows fast exploration of long-term trends, the identification of the dominant mechanisms, and screening of operational scenarios that would be impracticably expensive to simulate exhaustively with full-scale simulations by distilling system behavior into a limited set of physically interpretable parameters [50]. The low-order approach can also form a bridge to span the gap between conceptual thinking and numerical modeling. The insights provided from the dynamical-systems point of view have the potential to help identify parameters, design appropriate scenarios, and interpret results from high-fidelity models to make them more effective and transparent [51]. The current approach aims to develop this model as a conceptual and screening-level tool, complementing other geothermal modeling approaches rather than competing with them. Such multi-scale modeling strategies are increasingly seen as essential for managing complex subsurface energy systems under uncertainty.

7. Limitations and future work

The low-order dynamical model framework that has been developed in this work is modelled in such a way as to be a conceptual and interpretive tool. Although the above method has the advantage of providing insight and physical interpretation, it also has a limitation in its structure. First, the present representation lacks a description of the interaction between the processes considered thermal, hydraulic, or mechanical. In many geothermal environments, processes like permeability changes, deformation, or stress-mediated changes in flow pathways can impact on heat transport and thermal depletion on a large timescale [52]. An extension of this approach toward incorporating the effects of thermo-hydro-mechanical processes seems a natural next course for future development. Secondly, the model parameters represent effective properties that are considered time-invariant. These parameters do not require calibration based on geothermal reservoirs. Thus, the current study is not intended to make site-specific predictions. Site-calibrated parameters based on production rate and corresponding temperatures of already existing geothermal facilities would enable the assessment of parameter values, verification of the role of the time delay term, and analysis of model validity in a wide range of geological conditions [53]. This procedure is crucial for the incorporation of conceptual understanding at an application level. Homogeneous effective properties correspond to the approach where the lumped parameter models average spatial variations using effective properties. Homogeneous effective properties correspond to the approach where the lumped parameter models average spatial variations using effective properties. This implicitly assumes that the finite spatial variations may not be accounted for in the model. However, constant and ideal models based on production rate assumptions isolate the role of the thermal delay effect. Time-dependent production models are partially analyzed using the step function approach; however, more realistic approaches would be considered in future studies. Other processes such as the evolution of permeability based on thermochemical reactions like precipitation and scaling reactions have not been considered here because the focus of the study is based on energy balance.

Third, the present study does not specifically address uncertainty quantification. Uncertainty can arise through geological heterogeneity, generally sparse data in the subsurface, and long operational horizons. Probabilistic parameter estimation, sensitivity analysis, and stochastic forcing could be used to assess the robustness of delayed drawdown predictions to uncertainty in future studies [54]. Such analyses would be particularly useful in risk-informed decision making and in long-term planning. This model assumes priori-specialized production scenarios and neglects feedback between reservoir state and operational control. Incorporation of adaptive control-where extraction and reinjection rates are adjusted in real time in accord with inferred reservoir conditions-emerges as one promising direction for extension of this framework. Coupling the low-order model with control algorithms may allow real-time sustainability monitoring as well as adaptive reservoir management [55]. Although the current study focuses on conceptual understanding and qualitative dynamics, it forms the basis for a series of extensions that would be needed to develop low-order dynamical models of geothermal reservoirs into an operational engineering practice, including coupled physics, field validation, uncertainty quantification, and adaptive control.

8. Conclusion

This work has introduced the low-order dynamical framework to study the long-term thermal drawdown of geothermal reservoirs. Considering the reservoir as a slow thermal state variable driven by extraction and recharge processes, this model offers a physically describable interpretation of the reservoir-scale thermal evolution without recourse to high-fidelity spatial simulations. A central contribution of this study is the explicit incorporation of time-delayed thermal response within the governing formulation. This delay embodies the geologic memory characteristics of geothermal systems and furnishes a transparent rationale for the frequently observed pattern of early-time stability followed by rapid thermal drawdown. By rendering these delays explicit, the formulation offers a clear justification for why short-term operational data may understate long-term depletion risk. Both analytical and numerical results described here show how selected important features of geothermal reservoir behavior, such as the delayed temperature decline, sensitivity to sustained extraction rates, and long-term equilibrium trends, can be reproduced with a minimum of assumptions. Since the model is nonsite-specific, substantial insight is gained into the dominant mechanisms governing long-term sustainability. This should be considered a screening-level and educational tool, complementary to detailed numerical simulations. The current approach enables rapid exploration of long-term scenarios and improves conceptual insight into time-lagged thermal dynamics in systems with long characteristic time scales, while providing a foundation for future extensions involving calibration, uncertainty analysis, and higher-fidelity coupling. In this respect, the model fosters a more open and system-level view on geothermal reservoir sustainability. Comparisons of the proposed framework with long-term production data from operating geothermal fields are proposed as future work.

Ethical issue
The author is aware of and comply with best practices in publication ethics, specifically with regard to authorship (avoidance of guest authorship), dual submission, manipulation of figures, competing interests, and compliance with policies on research ethics. The author adheres to publication requirements that the submitted work is original and has not been published elsewhere.

Data availability statement
The manuscript contains all the data. However, more data will be available upon request from the authors.

Conflict of interest
The author declares no potential conflict of interest.

References

[1] Kassem, M.A., Moscariello, A. (2025). Geothermal energy: A sustainable and cost-effective alternative for clean energy production and climate change mitigation, Sustainable Futures, 10, 101247. https://doi.org/10.1016/j.sftr.2025.101247.

[2] de Bruijn, E.A.M., Bloemendal, M., ter Borgh, M.M., Godderij, R.R.G.G. , Vossepoel, F.C. (2021). Quantifying the contribution of heat recharge from confining layers to geothermal resources, Geothermics, 93, 102072. https://doi.org/10.1016/j.geothermics.2021.102072.

[3] White, M., Vasyliv, Y., Beckers, K., Martinez, M., Balestra, P., Parisi, C., Augustine, C., Bran-Anleu, G., Horne, R., Pauley, L., Bettin, G., Marshall, T., Bernat, A. (2024). Numerical investigation of closed-loop geothermal systems in deep geothermal reservoirs, Geothermics,116, 102852. https://doi.org/10.1016/j.geothermics.2023.102852.

[4] Reimus, P., Caporuscio, F., Marina, O., Janney, D. (2020). Field demonstration of the combined use of thermally-degrading and cation-exchanging tracers to predict thermal drawdown in a geothermal reservoir, Geothermics, 83, 101712, https://doi.org/10.1016/j.geothermics.2019.101712.

[5] Haklıdır, F.S.T. (2020). The importance of long-term well management in geothermal power systems using fuzzy control: A Western Anatolia (Turkey) case study, Energy, 213, 118817, https://doi.org/10.1016/j.energy.2020.118817.

[6] Gao, X., Li, T., Zhang, Y., Kong, X., & Meng, N. (2022). A Review of Simulation Models of Heat Extraction for a

Geothermal Reservoir in an Enhanced Geothermal System. Energies, 15(19), 7148. https://doi.org/10.3390/en15197148.
[7] Franco, A., & Vaccaro, M. (2014). Numerical simulation of geothermal reservoirs for the sustainable design of energy plants: A review. Renewable & Sustainable Energy Reviews, 30, 987–1002. https://doi.org/10.1016/J.RSER.2013.11.041.
[8] Fitzgerald, S. D., & Woods, A. W. (1998). Instabilities during liquid migration into superheated geothermal reservoirs. Water Resources Research, 34(9), 2089–2101. https://doi.org/10.1029/98WR01448.
[9] Zeedan, A., Abd, A., & Abushaikha, A. S. (2025). Reservoir Simulations: A Comparative Review of Machine Learning Approaches. IEEE Access, 1. https://doi.org/10.1109/access.2025.3614017.
[10] Barbier, E. (1997). Nature and technology of geothermal energy: A review. Renewable and Sustainable Energy Reviews, 1(1–2), 1–69. https://doi.org/10.1016/S1364-0321(97)00001-4.
[11] Qin, Z., Qin, Z., Valfells, A., & Guðjónsdóttir, M. S. (2017). The lumped-parameter model on two-phase and superheated geothermal reservoir. Energy Procedia, 142, 481–487. https://doi.org/10.1016/J.EGYPRO.2017.12.075.
[12] Omrani, P.S., Egberts, P. J. P., Rijnaarts, H. H. M., & Shariat Torbaghan, S. (2026). Geothermal plant operation and control under demand uncertainties. Renewable Energy, 257, 124805. https://doi.org/10.1016/j.renene.2025.124805.
[13] Fan, R., Jiang, Y., Yao, Y., Shiming, D., & Ma, Z. (2007). A study on the performance of a geothermal heat exchanger under coupled heat conduction and groundwater advection. Energy, 32(11), 2199–2209. https://doi.org/10.1016/j.energy.2007.05.001.
[14] Millstein, D., Dobson, P., & Jeong, S. (2021). The Potential to Improve the Value of U.S. Geothermal Electricity Generation Through Flexible Operations Open Access. J. Energy Resour. Technol., 143(1), 010905. https://doi.org/10.1115/1.4048981.
[15] Ostermann, I. (2011). Three-dimensional modeling of heat transport in deep hydrothermal reservoirs. Gem - International Journal on Geomathematics, 2(1), 37–68. https://doi.org/10.1007/S13137-011-0017-Y.
[16] Pandey, S. N., & Singh, M. (2021). Artificial Neural Network to Predict the Thermal Drawdown of Enhanced Geothermal System Available to Purchase. J. Energy Resour. Technol., 143(1), 010901. https://doi.org/10.1115/1.4048067.
[17] Abdullah, F., Wu, Z., & Christofides, P. D. (2021). Data-based reduced-order modeling of nonlinear two-time-scale processes. Chemical Engineering Research and Design, 166, 1–9. https://doi.org/10.1016/j.cherd.2020.11.009.
[18] Liu, PH., & Lin, JC. (2025). Integrated risk assessment and mitigation strategies for geothermal energy development: technical, socio-political, and financial dimensions. Sustainable Energy Res., 12, 64. https://doi.org/10.1186/s40807-025-00218-3.
[19] Liu, S., Chai, J., Liu, J., & Xue, Y. (2024). Effect of water-cooling shock on heat production performance of enhanced geothermal systems. Energy Sources, Part A: Recovery, Utilization, and Environmental Effects, 46(1), 559–577. https://doi.org/10.1080/15567036.2023.2284843.
[20] Yang, F., Wang, G., Hu, D., Zhou, H., & Tan, X. (2022). Influence of water-rock interaction on permeability and heat conductivity of granite under high temperature and pressure conditions. Geothermics, 100, 102347. https://doi.org/10.1016/j.geothermics.2022.102347.
[21] Ciriello, V., Bottarelli, M., Di Federico, V., & Tartakovsky, D. M. (2015). Temperature fields induced by geothermal devices. Energy, 93(Part 2), 1896–1903. https://doi.org/10.1016/j.energy.2015.10.052.
[22] Bagalkot, N., & Kumar, G. S. (2015). Thermal front propagation in variable aperture fracture–matrix system: A numerical study. Sadhana, 40, 605–622. https://doi.org/10.1007/s12046-014-0324-8.
[23] Heinze, T., Hamidi, S., & Galvan, B. (2017). A dynamic heat transfer coefficient between fractured rock and flowing fluid. Geothermics, 65, 10–16. https://doi.org/10.1016/j.geothermics.2016.08.007.
[24] Khan, Z. H., Zhou, M., Trounev, A., & Khan, W. A. (2025). Modeling liquid–vapor fronts in porous media using time-fractional derivatives: An innovative framework. Physics of Fluids, 37, 032112. https://doi.org/10.1063/5.0259241.
[25] Gan, Q., & Elsworth, D. (2014). Analysis of fluid injection-induced fault reactivation and seismic slip in geothermal reservoirs. J. Geophys. Res. Solid Earth, 119, 3340–3353. https://doi.org/10.1002/2013JB010679.
[26] Sarak, H., Onur, M., & Satman, A. (2005). Lumped-parameter models for low-temperature geothermal fields and their application. Geothermics, 34(6), 728–755. https://doi.org/10.1016/j.geothermics.2005.09.001.
[27] Singh, M., Mahmoodpour, S., Schmidt-Hattenberger, C., Sass, I., & Drews, M. (2024). Influence of Reservoir Heterogeneity on Simultaneous Geothermal Energy Extraction and CO2 Storage. Sustainability, 16(1), 387. https://doi.org/10.3390/su16010387.
[28] Mudunuru, M. K., Karra, S., Harp, D. R., Guthrie, G. D., & Viswanathan, H. S. (2017). Regression-based reduced-order models to predict transient thermal output for enhanced geothermal systems. Geothermics, 70, 192–205. https://doi.org/10.1016/j.geothermics.2017.06.013.
[29] Bujakowski, W., Tomaszewska, B., & Miecznik, M. (2016). The Podhale geothermal reservoir simulation for long-term sustainable production. Renewable Energy, 99, 420–430. https://doi.org/10.1016/j.renene.2016.07.028.
[30] Goodwin, N., May, M., Wilson, S., & Graham, G. (2020). Mineral Solubility Measurement and Prediction Under High Pressure, High Temperature Reservoir Conditions. https://doi.org/10.2118/199285-MS.
[31] Steingrimsson, B., Bodvarsson, G. S., Cuellar, G., & Escobar, C. (1989). Changes in thermodynamic conditions of the Ahuachapan reservoir due to

production and injection. https://digital.library.unt.edu/ark:/67531/metadc1198144/m2/1/high_res_d/6680654.pdf.
[32] Mégel, T., & Rybach, L. (2000). Production capacity and sustainability of geothermal doublets. Proceedings World Geothermal Congress 2000. Kyushu - Tohoku, Japan, May 28 - June 10, 2000
[33] Chevarunotai, N., Hasan, A. R., Kabir, C. S., & Islam, R. (2018). Transient flowing-fluid temperature modeling in reservoirs with large drawdowns. Journal of Petroleum Exploration and Production Technology, 8(3), 799–811. https://doi.org/https://doi.org/10.1007/s13202-017-0397-0.
[34] Hermanns, M., & Pérez, J. M. (2014). Asymptotic Analysis of Vertical Geothermal Boreholes in the Limit of Slowly Varying Heat Injection Rates. Siam Journal on Applied Mathematics, 74(1), 60–82. https://doi.org/10.1137/130930170.
[35] Maar, L., & Seifermann, S. (2023). Assessing the Environmental Sustainability of Deep Geothermal Heat Plants. Energies, 16(19), 6774. https://doi.org/10.3390/en16196774.
[36] Wang, Y., Karimi, H. R., & Xiang, Z. (2013). Delay-Dependent Control for Networked Control Systems with Large Delays. Mathematical Problems in Engineering, 2013, 1–10. https://doi.org/10.1155/2013/643174.
[37] Yang, Y., Fu, G., Zhao, J., & Gu, L. (2023). Heat Production Capacity Simulation and Parameter Sensitivity Analysis in the Process of Thermal Reservoir Development. Energies. https://doi.org/10.3390/en16217258.
[38] Sekar, L. K., & Okoroafor, E. R. (2024). Maximizing Geothermal Energy Recovery from Enhanced Geothermal Systems Through Huff-And-Puff: A Comprehensive Simulation Study. https://doi.org/10.56952/arma-2024-0177.
[39] Gan, Q., & Elsworth, D. (2014). Thermal drawdown and late-stage seismic-slip fault reactivation in enhanced geothermal reservoirs. Journal of Geophysical Research, 119(12), 8936–8949. https://doi.org/10.1002/2014JB011323.
[40] Kikuchi, C. P., & Ferré, T. P. A. (2017). Analysis of subsurface temperature data to quantify groundwater recharge rates in a closed Altiplano basin, northern Chile. Hydrogeology Journal, 25(1), 103–121. https://doi.org/10.1007/S10040-016-1472-1.
[41] Gu, X., Makasis, N., Motamedi, Y., Narsilio, G. A., Arulrajah, A., & Horpibulsuk, S. (2021). Geothermal pavements: field observations, numerical modelling and long-term performance. Geotechnique, 72(9), 832–846. https://doi.org/10.1680/jgeot.20.P.296.
[42] Gudala, M., Tariq, Z., Govindarajan, S., Yan, B., & Sun, S. (2024). Fractured Geothermal Reservoir Using CO2 as Geofluid: Numerical Analysis and Machine Learning Modeling. ACS Omega. https://doi.org/10.1021/acsomega.3c07215.
[43] De Simone, S., De Simone, S., Vilarrasa, V., Vilarrasa, V., Carrera, J., Alcolea, A., & Meier, P. M. (2013). Thermal coupling may control mechanical stability of geothermal reservoirs during cold water injection. Physics and Chemistry of The Earth, 64, 117–126. https://doi.org/10.1016/J.PCE.2013.01.001.
[44] Pinyol, N. M., Alonso, E., Corominas, J., & Moya, J. (2012). Canelles landslide: modelling rapid drawdown and fast potential sliding. Landslides, 9(1), 33–51. https://doi.org/10.1007/S10346-011-0264-X.
[45] Magicho, D. O., & Kwasira, J. (2025). Risk Dynamics and Performance of Geothermal Drilling Projects in Menengai, Nakuru County, Kenya. International Journal of Social Science and Humanities Research, 3(2). https://doi.org/10.61108/ijsshr.v3i2.207.
[46] Li, M., Zhang, S., Zhao, Y., Cai, Y., Zhang, M., Liu, Z., Li, P., Wang, B., Xu, B., Shen, J., & Feng, B. (2025). Multifactor Coupling Effects on Permeability Evolution During Reinjection in Sandstone Geothermal Reservoirs: Insights from Dynamic Core Flow Experiments. Energies, 18(17), 4770. https://doi.org/10.3390/en18174770.
[47] Hamid, O., & Abdalrahman, E. M. (2024). Renewable Management Strategies for Geothermal Energy Reservoirs. https://doi.org/10.56952/igs-2024-0872.
[48] Robinson, B., Cohen, J. S., & Herman, J. D. (2020). Detecting early warning signals of long-term water supply vulnerability using machine learning. Environmental Modelling and Software, 131, 104781. https://doi.org/10.1016/J.ENVSOFT.2020.104781.
[49] Podgorney, R., Huang, H., Plummer, M., & Gaston, D. (2013). A Globally-Implicit Computational Framework for Physics-based Simulation of Coupled Thermo-Hydro-Mechanical Problems: Application to Sustainability of Geothermal Reservoirs. 23(1), 51–58. https://doi.org/10.11128/SNE.23.TN.10175.
[50] Pasetto, D., Pasetto, D., Guadagnini, A., & Putti, M. (2014). A reduced-order model for Monte Carlo simulations of stochastic groundwater flow. Computational Geosciences, 18(2), 157–169. https://doi.org/10.1007/S10596-013-9389-4.
[51] Marino, R., Giambagli, L., Chicchi, L., Buffoni, L., & Fanelli, D. (2023). A Bridge between Dynamical Systems and Machine Learning: Engineered Ordinary Differential Equations as Classification Algorithm (EODECA). arXiv.Org, abs/2311.10387. https://doi.org/10.48550/arxiv.2311.10387.
[52] Zhang, G. Y. (2024). Optimizing high-temperature geothermal extraction through THM coupling: insights from SC-CO2 enhanced modeling. Engineering Computations. https://doi.org/10.1108/ec-11-2023-0889.
[53] Muqtadir, A., Zhang, B., Ben Saleh, W., Chalaturnyk, R., Vaisblat, N., Wigston, A., Drobot, A., Groenewoud, L., & Kirsten, M. (2025). Geothermal reservoir modeling and calibration in a coupled thermo-hydro-mechanical perspective - a case study for DEEP Geothermal Project in Saskatchewan, Canada. 59th U.S. Rock Mechanics/Geomechanics Symposium. https://doi.org/10.56952/ARMA-2025-0617.

[54] Pyrcz, M. J., & White, C. D. (2015). Uncertainty in reservoir modeling. Interpretation, 3(2). https://doi.org/10.1190/INT-2014-0126.1.

[55] Noye, S., Mulero Martinez, R., Carnieletto, L., De Carli, M., & Castelruiz Aguirre, A. (2022). A review of advanced ground source heat pump control: Artificial intelligence for autonomous and adaptive control. Renewable & Sustainable Energy Reviews, 153, 111685. https://doi.org/10.1016/J.RSER.2021.111685.