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The application of the Bayesian linear regression model to optimize the maintenance of a programmable logic controller

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ABSTRACT

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1. Introduction

Programmable logic controllers (PLCs), which are essential components of industrial automation systems, are used in manufacturing settings to manage and monitor a variety of operations. For PLCs, effective maintenance procedures are critical to ensuring their longevity and reliability. Although several authors have proposed and developed models for the management of maintenance, longevity, and reliability of the PLC systems [1-5]; however, there are still limited studies, on the specific subjects of maintenance optimization for PLC systems. Through typical maintenance optimization, organizations can reduce the probability of unexpected failures, such that they can reduce the amount of downtime and increase productivity. Also, they will be able to guarantee that PLC systems run at optimal efficiency with the lowest failure risk by optimizing maintenance schedules based on criteria such as equipment usage, operational conditions, and historical performance data. This method bridges the gap between academic research and the practical application of maintenance optimization models. Dekker [6] described his perception of an optimization model as representing a technical system, its function and importance, system deterioration, available

system information, an objective function, and an optimization technique. Wang [5] created a general framework for optimizing maintenance policies, system configuration, maintenance effectiveness, maintenance cost, optimization criteria, modeling tools, planning horizon, reliability, and system information which are used as inputs for the framework. According to Marais & Saleh [7], different optimization models can be obtained by changing the system configuration, maintenance effectiveness, planning horizon, analytical tools, and component dependencies. Although this provides a good idea for building a maintenance optimization model; however, it does not include all of the optimization classes. Optimization classes are the input parameters required to build a maintenance optimization model, which is expected to produce the desired output. While most optimization models adopt a component perspective, Tan & Raghavan [8] developed a framework for a predictive maintenance-based plan generated from a system perspective. Wang [5] and Nicolai & Dekker [3] considered the planning horizon when categorizing the different optimization methods where they were classified into models for finite periods; however, they didn't consider the exploration of maintenance optimization. Using the Markov

In this paper, an optimization approach, which is based on the Bayesian Linear

Inference (BLI) model, has been proposed for the maintenance of Programmable Logic Controllers (PLCs). The BLI model, which is implemented

using historical data, incorporates maintenance indicators like the number of

failures (NF), total downtime (TD), total unexpected intervals (TUI), mean time to repair (MTTR) and mean time between failures (MTBF). It offers a

probabilistic framework for determining the influence of each predictor

variable on PLC maintenance. The model produces posterior means, credible

intervals, and standard deviations, which provide insights into the magnitude

and uncertainty of these relationships. The results from the study show that

factors like NF and TD are influenced by the magnitude and direction of the maintenance levels. Also, the R-squared score (0.85) also indicates how much

of the variability in maintenance in the system. From the results obtained, the study can conclude that the BLI model can optimize PLC maintenance

procedures by identifying essential components and their contributions. Also, it is able to estimate future maintenance requirements and helps with resource

allocation and process optimization decisions.

analysis method, Alizadeh & Sriramula [9] and Liu & Frangopol [10] presented a novel reliability model for redundant safety-related systems. Providing a logical reliability assessment of ship structures under various threats throughout their lifecycle. A flexible set of modeling patterns was presented by Meng et al. [11] and implemented in the Alta-Rica 3.0 language. Chen & Mehrabani [12] introduced a technique for analyzing the reliability of coastal flood defenses, such as earth sea dykes, about changing operating conditions. The method also included future performance projections and the best maintenance plan. A unique approach to reliability-centered maintenance based on artificial neural networks was introduced by Pliego Marugán et al. [13]. Zhu et al. [14] presented and examined a reliability and maintenance model of a k-out-of-n: F system for PLCs. During this process, the system underwent a rebuilding process with reduced performance, which was followed by preventive maintenance (PM) with the replacement of malfunctioning components. During this rebuilding process, the system was susceptible to failure with various failure criteria. Izquierdo et al. [15] proposed a novel strategy that used a case study approach to validate it, which helped to reduce the uncertainty arising from the operational context. A condition-based maintenance decision framework for a multi-component system subject to a system reliability requirement was created by Shi et al. [16]. Ma et al. [17] looked into the methodologies for maintenance optimization and reliability analysis of a two-unit warm standby cooling system. A performance-balanced system operating in a shock environment was proposed by Wang et al. [18], which is hardly observed in the literature. The joint optimization of lot sizing and maintenance policy for a multi-product production system subject to two failure scenarios was studied by Gao et al. [19]. Chang et al. [20] applied the approach of minimal cuts for demand d (d-MC) to evaluate the time-related reliability of a multi-state flow network (MSFN).

To address the maintenance optimization issue of the PLCs system, a Bayesian Linear Inference (BLI) model has been proposed in this study. The BLI is a potent statistical method that maximizes maintenance strategies by utilizing both linear modeling and Bayesian principles. Using the BLI model to schedule and carry out maintenance for PLC systems transforms the process and results in lower costs, downtime, and an increase in the system's reliability. With the BLI model, the study will be able to take into account system variability and uncertainties, which are common within the complex and dynamic environments in which PLCs operate. This model is especially useful as it is possible to accurately estimate future maintenance requirements by simulating the interactions between the many elements that characterize the PLCs system performance. Maintenance workers can make wellinformed decisions and judgments using these approaches, which offer a probabilistic framework based on the likelihood of various outcomes and for handling the inherent uncertainties in PLC's behavior, such as wear and tear, weather conditions, and component deterioration.

2. Bayesian linear inference model

The Bayesian Linear Inference model is a probabilistic version of the linear regression that applies the Bayesian principles. It represents a framework for the estimation of the parameters of a linear regression model taking into account the uncertainty, and allowing for re-use of previous experience. It is a powerful agent in incorporating past knowledge of the parameters. This is most beneficial in cases where such information about the variables or parameters is available previously. Uncertainty in parameter estimations is captured in it. Rather than providing point estimates of regression coefficients and error variance, the model generates posterior distributions that represent the range of feasible values for the parameters in light of the observed data and previous knowledge. The governing equations of the model have been presented in the following definitions.

3. Definition

Let the prior distribution of θ be given as a normal distribution $N(\mu,\Sigma)$ where μ is the mean and can also be referred to as the first moment and Σ be the covariance matrix and the second moment of the distribution, such that the probability of θ is given as:

$$P(\theta) = \frac{1}{z} exp\left\{-\frac{1}{2}(\theta - \mu)^T \Sigma^{-1}(\theta - \mu)\right\}$$
(1)

where

$$\mu = E_{p(\theta)}[\theta]$$
 and $\Sigma = E_{p(\theta)}[(\theta - \mu)^T(\theta - \mu)]$

Equation (1) is the governing equation of the BLI model, and it refers to the moment parametrization of θ since it consists of the first moment (μ) and the second moment (Σ) of the variable. Z is a normalization factor with the value $\sqrt{(2\pi)^n \det(\Sigma)}$, where,*n* is the dimension of θ . To prove this equation, one can translate the distribution from the origin and do a change of variables such that the distribution has the form and can be expressed θ' in polar coordinates and integrate over the space to compute Z.

$$P(\theta') = \frac{1}{Z} exp\left\{-\frac{1}{2}\theta'^{T}\theta'\right\}$$
(2)

With the BLI model, it is possible to determine the probability of an output y_{t+1} given a new input x_{t+1} and the set of data $D = \{(x_i, y_i)\}_i = 1, \dots, t$. To compute the probability $P(y_{t+1}|x_{t+1}, D)$, the distribution θ is introduced into this expression and marginalize over it.

$$P(y_{t+1}|x_{t+1}, D) = \int_{\theta \in \Theta} P(y_{t+1}|x_{t+1}, \theta, D) P(\theta|x_{t+1}, D)$$
(3)

D explains no more than what θ does, $P(y_{t+1}|x_{t+1}, \theta, D)$ is essentially $P(y_{t+1}|x_{t+1}, \theta,)$. Also, from the graphical model the study can determine $P(\theta|x_i, D)$ is $P(\theta|D)$ since y_i is known and thus θ and x_i are independent, hence, Equation (3) can be rewritten as:

$$P(y_{t+1}|x_{t+1}, D) = \int_{\theta \in \Theta} P(y_{t+1}|x_{t+1}, \theta) P(\theta|D)$$
(4)

However, computing with the above equation may be too complex due the moment parameterization of normal distributions(θ) but not with the natural parameterization. Hence, the moment parameterization of normal distributions(θ) is converted to natural parameterization of normal distributions in the form $P(x) = \frac{1}{z} exp \left\{ -\frac{1}{2}(x - \mu)^T \Sigma^{-1}(x - \mu) \right\}$ which can also be expressed further as:

$$P(x) = \frac{1}{z} exp\left\{J^T x - \frac{1}{2} x^T \check{P} x\right\}$$
(5)

The natural parameterization simplifies the multiplication of normal distributions as it becomes the addition of the *J* and \check{P} matrices of different distributions. Transforming the moment parameterization to the natural parameterizationis done by first expanding the exponent:

$$-\frac{1}{2}(x-\mu)^{T}\Sigma^{-1}(x-\mu) = -\frac{1}{2}x^{T}\Sigma^{-1} + \mu^{T}\Sigma^{-1}x - \frac{1}{2}\mu^{T}\Sigma^{-1}\mu$$
(6)

The last term in the above equation, has nothing to do with x and can therefore be absorbed into the normalizer, by comparing equations (5) and (6), J and \tilde{P} therefore can be expressed as:

$$\begin{cases} J = \Sigma^{-1} \mu \\ \check{P} = \Sigma^{-1} \end{cases}$$
(7)

where the matrix \check{P} is called the precision matrix.

4. Posterior distribution $P(\theta|D)$

Using Bayes rule, the posterior probability $P(\theta|D)$ can be expressed as

$$P(\theta|D) \propto P(y_{1:t}|x_{1:t},\theta)P(\theta) \propto (\prod_{i=1}^{t} P(y_1|x_1,\theta))P(\theta)$$
(8)

The y_i 's and θ have a diverging relationship at θ , and since θ is unknown, it follows that the y_i 's are independent of each other; that is, $P(y_{1:t}|x_{1:t},\theta) = \prod_{i=1}^{t} P(y_1|x_1,\theta)$. An easy updating rule can compute this product. By examining the result of $P(y_1|x_1,\theta)P(\theta)$.

$$\begin{split} P(y_1|x_1,\theta)P(\theta) &\propto exp\left\{-\frac{1}{2\sigma^2}(y_i-\theta^T x)^2\right\}exp\left\{J^T-\frac{1}{2}\theta^T P\theta\right\} &\propto exp\left\{-\frac{1}{2\sigma^2}(-2y_i\theta^T x_i+\theta^T x_i x_i^T\theta)\right\}exp\left\{J^T\theta-\frac{1}{2}\theta^T P\theta\right\} &= exp\left\{\frac{1}{\sigma^2}y_i x^T\theta-\frac{1}{2\sigma^2}\theta^T x_i x_i^T\theta\right\}exp\left\{J^T\theta-\frac{1}{2}\theta^T P\theta\right\} &= exp\left\{\left(J+\frac{1}{\sigma^2}y_i x_i\right)^T\theta-\frac{1}{2}\theta^T\left(P+\frac{1}{\sigma^2}x_i x_i^T\right)\theta\right\} \\ &= exp\left\{J'^T\theta-\frac{1}{2}\theta^T P'\theta\right\} \end{split}$$

where $P(y_1|x_1, \theta)$ is the likelihood function and $P(\theta)$ is the prior distribution. The equation was broken down to understand the components of the posterior distribution as.

i.
$$P(y_1|x_1,\theta)P(\theta) \propto exp\left\{-\frac{1}{2\sigma^2}(y_i-\theta^T x)^2\right\}exp\left\{J^T-\frac{1}{2}\theta^T P\theta\right\}$$

This step involves multiplying the likelihood function $P(y_1|x_1,\theta)$ and the prior distribution $P(\theta)$ together. The likelihood function represents the probability of observing the data y_1 given the parameters θ and x_1 . The prior distribution represents our initial beliefs about the distribution of θ before observing any data.

ii.
$$\propto exp\left\{-\frac{1}{2\sigma^2}\left(-2y_i\theta^Tx_i+\theta^Tx_ix_i^T\theta\right)\right\}exp\left\{J^T\theta-\frac{1}{2}\theta^TP\theta\right\}$$

In this step, the quadratic term was expanded in the exponential and the expression was simplified. The terms were combined with θ to form a quadratic form.

iii.
$$\propto exp\left\{\frac{1}{\sigma^2}y_ix^T\theta - \frac{1}{2\sigma^2}\theta^Tx_ix_i^T\theta\right\}exp\left\{J^T\theta - \frac{1}{2}\theta^TP\theta\right\}$$

Here, the terms were collected with θ to rewrite the expression.

iv.
$$\propto exp\left\{J'^T\theta - \frac{1}{2}\theta^T P'\theta\right\}$$

Finally, the terms involving θ were combined, resulting in the desired form. *J*' and *P*' represent new vectors or matrices obtained from the original terms, depending on the values of *J*, *P*, *y*_{*i*}, *x*_{*i*}, and σ respectively.

The resulting expression is proportional to the exponential of a quadratic form in θ . This form is typical in Bayesian inference, where the posterior distribution is often proportional to the exponential of a quadratic form due to the conjugacy of certain prior and likelihood combinations. Line 1 to line 2 is true because any term that does not have θ can be absorbed into the normalizer. Now, we can apply the generalized result to Equation (8) and derive the following:

$$P(\theta|D) \propto exp\left\{ \left(J + \frac{\sum_{i} y_{i} x_{i}}{\sigma^{2}} \right)^{T} \theta - \frac{1}{2} \theta^{T} \left(P + \frac{\sum_{i} x_{i} x_{i}^{T}}{\sigma^{2}} \right) \right\}$$
(9)

where, $P(\theta|D)$ is a normal distribution with $J_{final} = J + \frac{\sum_i v_i x_i}{\sigma^2}$ and $P_{final} = P + \frac{\sum_i x_i x_i^T}{\sigma^2}$. P_{final} is the precision matrix of the normal distribution, and as the number of x_i increases, the terms in this matrix become larger. Also, since P_{final} is the inverse of the covariance, the variance gets lower as the number of samples grows. This is a characteristic of a Gaussian model that a new data point always lowers the variance, but this downgrading of variance does not always make sense. If we believe that there are outliers in our dataset, this model will not work. With the relation previously given, the mean and covariance of this distribution may be determined:

$$\mu_{final} = \left(\Sigma^{-1} + \frac{\sum_{i} x_{i} x_{i}^{T}}{\sigma^{2}}\right)^{-1} \frac{\sum_{i} y_{i} x_{i}}{\sigma^{2}}$$

$$\Sigma_{final} = \left(\Sigma^{-1} + \frac{\sum_i x_i x_i^T}{\sigma^2}\right)^{-1}$$

where, μ_{final} is the mean of the distribution and Σ_{final} is the covariance of the distribution.

 μ_{final} and Σ_{final} are broken down as:

$$\mu_{final} = \left(\Sigma^{-1} + \frac{\sum_{i} x_{i} x_{i}^{T}}{\sigma^{2}}\right)^{-1} \frac{\sum_{i} y_{i} x_{i}}{\sigma^{2}}$$

In this Equation, Σ represents the covariance matrix and Σ^{-1} denotes its inverse. The term $(\sum_i x_i x_i^T)$ represents the sum of the outer products of the input vectors x_i . σ is the standard deviation or noise parameter. The expression $(\sum_i y_i x_i)$ represents the sum of the product of the observed target values y_i and the corresponding input vectors x_i . The expression calculates the updated value of the mean parameter μ_{final} . It involves matrix computations where the inverse of the covariance matrix Σ is added to the sum of the outer products $(\sum_i x_i x_i^T)$. This sum of outer products captures the structure of the input data. The term $(\sum_i y_i x_i)$ is multiplied by the inverse of the noise parameter σ^2 .

i.
$$\Sigma_{final} = \left(\Sigma^{-1} + \frac{\sum_i x_i x_i^T}{\sigma^2}\right)^{-1}$$

Here, the expression calculates the updated value of the covariance matrix Σ_{final} . It involves a similar matrix computation as in the previous Equation. The inverse of the covariance matrix Σ^{-1} is added to the sum of the outer products ($\sum_i x_i x_i^T$), capturing the structure of the input data. This sum is then inverted to obtain the updated covariance matrix Σ_{final} .

These Equations are used in BLI to update the mean and covariance of the posterior distribution of the parameters. They incorporate the observed data and provide a way to update the prior beliefs based on the likelihood of the data and the noise parameter σ .

5. Probability distribution of the prediction

The next step is to compute the probability distribution of prediction $P(y_{t+1}|x_{t+1}, \theta)$. Since the linear combination of normal distributions is also a normal distribution,

$$\begin{split} P(y_{t+1}|x_{t+1},\theta) \text{therefore the distribution can be written in the} \\ \text{form} \frac{1}{z} exp\left\{-\frac{1}{2\sigma^2} (y_{t+1} - \mu_{y_{t+1}})^T \Sigma_{y_{t+1}} (y_{t+1} - \mu_{y_{t+1}})\right\}, \text{ where} \\ \mu_{y_{t+1}} &= E[y_{t+1}] = E[\theta^T x_{t+1} + \epsilon] = E[\theta^T x_{t+1}] + E[\epsilon] = \\ E[\theta]^T x_{t+1} + 0 = \mu_{\theta}^T x_{t+1} \\ \text{and} \end{split}$$

 $\Sigma_{y_{t+1}} = x_{t+1}^T \Sigma_{\theta} x_{t+1} + \sigma^2$

The components are broken down and explained as:

i.
$$P(y_{t+1}|x_{t+1},\theta) = \frac{1}{z}exp\left\{-\frac{1}{2\sigma^2}(y_{t+1}-\mu_{y_{t+1}})^T \Sigma_{y_{t+1}}(y_{t+1}-\mu_{y_{t+1}})\right\}$$

This equation represents the conditional probability distribution of the target variable y_{t+1} given the input variable x_{t+1} and the parameter θ . It is characterized by a multivariate Gaussian distribution.

ii.
$$\mu_{y_{t+1}} = E[y_{t+1}] = E[\theta^T x_{t+1} + \epsilon] = E[\theta^T x_{t+1}] + E[\epsilon]$$

= $E[\theta]^T x_{t+1} + 0 = \mu_{\theta}^T x_{t+1}$

In this expression, $\mu_{y_{t+1}}$ represents the mean of the target variable y_{t+1} . It is calculated by taking the expected value of $\theta^T x_{t+1}$ and considering that the expected value of the noise term ϵ is zero. Thus, the mean of y_{t+1} is given by the dot product of the expected value of θ (denoted as μ_{θ}) and the input variable x_{t+1} .

iii.
$$\Sigma_{y_{t+1}} = x_{t+1}^T \Sigma_{\theta} x_{t+1} + \sigma^2$$

Here, $\Sigma_{y_{t+1}}$ represents the covariance matrix of the target variable y_{t+1} . It is calculated by taking the outer product of x_{t+1} and Σ_{θ} (the covariance matrix of θ) and adding the variance σ^2 . The expression captures the uncertainty in the target variable y_{t+1} based on the uncertainty in the parameter θ (represented by Σ_{θ}) and the noise level σ . In addition, the equation defines the conditional probability distribution of y_{t+1} given x_{t+1} and θ as a multivariate Gaussian distribution, characterized by the mean $\mu_{y_{t+1}}$ and covariance matrix $\Sigma_{y_{t+1}}$. These parameters depend on the expected value of θ (μ_{θ}), the input variable x_{t+1} , and the covariance matrix of θ (Σ_{θ}), as well as the noise level σ .

6. Application of the model, results and discussions

The summary output of the BLI model offers details on credible intervals, the posterior distribution of the coefficients, and other pertinent statistics. BLI yields a distribution for every coefficient rather than point estimates. The range of values that a coefficient is most likely to fall into with a given probability is represented by credible intervals. The linear regression model's details, such as coefficients, pvalues, R-squared, etc., are shown in the summary output. A coefficient shows how each predictor, and the dependent variable are related to one another. The importance of every prediction is shown by the P-value. Indicators of statistical significance have a low p-value (< 0.05). Alongside the fitted linear regression line are the real data points in this graphic. Due to the dependent variable's linear relationship to the predictors, the anticipated values are shown by the linear regression line. The places where the model might not fit well are indicated by data points deviating from the line. The distribution of residuals, or the disparities between actual and expected values, is displayed in the residuals plot. The residuals should ideally be dispersed randomly at about zero.

3.1 PLC maintenance

Number of Failures, or NF: Understanding the correlation between the number of failures and the months can be aided by linear regression. The NF plot has a positive relationship; although the data point have a good fit with the regression line. The data point suggests that failure rates have increased in the last few months. Residual plot exhibit a normal distribution with a negative (-ve) intercept at the y-axis.

Mean time to repair (MTTR): The MTTR plot has a positive relationship, with the data point having a good fit with the regression line. The data point suggests that failure rates have decreased in the last few months. Residual plot exhibit a normal distribution with a positive (+ve) intercept at the y-axis.

Total Downtime (TD): The TD plot has a positive relationship, with the data point having a good fit with the regression line. The data point suggests that failure rates have decreased in the last few months. Residual plot exhibit a normal distribution with a negative (-ve) intercept at the y-axis.

Total Unscheduled Incidents (TUI): The TUI plot has a negative relationship, with the data point having a good fit with the regression line. The data point suggests that failure rates have decreased in the last few months. Residual plot exhibit a normal distribution with a negative (-ve) intercept at the y-axis.

Mean time before failure (MTBF): The MTBF plot has a negative relationship, with the data point having a good fit with the regression line. The data point suggests that failure rates have decreased in the last few months. Residual plot exhibit a normal distribution with a positive (+ve) intercept at the y-axis.

Generally, the relationship between the predictor variables (NF, TD, TUI, MTTR, and MTBF) is revealed by the results from the implementation of the BLI model. A more detailed explanation of the essential elements and data used in the model are summary as follows:

a. The initial maintenance value is from January when all predictors are 0.0 and a 95% credible interval ([145.5, 154.9]) which indicates that the true value of the intercept is most likely located within this range about 95% of the time.

b. The modest p-value is <0.001, it is concluded that there is a significant difference between the intercept and zero. SD (2.1): The degree of fluctuation or uncertainty surrounding the estimate. NF (Number of Failures): A rise in failures of one unit is correlated with a rise in maintenance of 3.5 units. [2.8, 4.2] is the 95% credible interval for NF.SD (0.6): The estimate's level of uncertainty. Given the modest p-value (<0.001), a significant positive connection is implied. TD (Total Downtime): There is a -1.2 unit drop in maintenance for every unit rise in total downtime. Lower maintenance appears to be linked to increased total downtime, as indicated by the negative coefficient. [-1.7, -0.7] is the 95% credible interval for TD. There is a substantial negative association, as indicated by the p-value of 0.012. Similar interpretations as NF and TD apply to TUI (Total Unexpected Intervals), MTTR (Mean Time to Repair), and MTBF (Mean Time Between Failures). R-squared: With an Rsquared of 0.85, the model accounts for 85% of the variation in the maintenance measures. This gives a very good regression performance. In conclusion, estimates of each predictor's influence on PLC maintenance are provided by the Bayesian linear regression model, coupled with an explanation of the uncertainties surrounding these values (Table 1). It gives an indication of the factors that are highly correlated with maintenance and a gauge of how well the

model matches the data. These observations can be helpful in maximizing PLC's general maintenance plans (Figures 1-5).



Figure 1. Linear regression plot of number of failures



Figure 2. Linear regression plot of mean time to repair



Figure 3. Linear regression plot of total downtime



Figure 4. Linear regression plot of total unscheduled incidents



Figure 5. Linear regression plot of mean time before failure

Table 1. Bayesian linear inference results

Variable	Posterior mean	Credible interval	Posterior SD	Р
Intercept	150.2	[145.5, 154.9]	2.1	<0.01
NF	3.5	[2.8, 4.2]	0.6	< 0.01
TD	-1.2	[-1.7, -0.7]	0.3	0.012
TUI	0.02	[-0.1, 0.14]	0.08	0.775
MTTR	-5.8	[-0.72, -4.4]	1.2	<0.01
MTBF	0.15	[0.08, 0.22]	0.03	< 0.01

7. Conclusion

The use of Bayesian linear regression to optimize the overall maintenance of Programmable Logic Controllers (PLCs) is a valuable and insightful method. The resulting plots, which demonstrate the correlations between key maintenance indicators and overall maintenance levels, are a visual depiction of the model's predictions and uncertainty. The Bayesian linear regression model, with its posterior means, credible intervals, and standard deviations, allows for a more nuanced understanding of the impact of variables like the number of failures (NF), total downtime (TD), total unexpected intervals (TUI), mean time to repair (MTTR), and mean time between failures (MTBF) on PLC maintenance. These insights enable stakeholders to identify crucial factors impacting maintenance levels and make sound decisions about resource allocation and process improvement. The Rsquared values, which indicate the model's explanatory power, show what percentage of variability in maintenance the model explains. This fit value gives confidence that the model can reflect the complexity of the predictormaintenance connection. Furthermore, the provided Bayesian linear regression model provides a probabilistic framework that takes into account uncertainty in parameter estimations, which improves its robustness in real-world applications. This functionality is especially important in the dynamic and frequently unpredictable industrial settings where PLCs operate. The application of Bayesian linear regression to optimize PLC maintenance demonstrates a thorough and adaptable methodology. This approach, which uses historical data and probabilistic modelling, provides a valuable tool for not just anticipating future maintenance requirements, but also strategically improving overall system reliability and efficiency. The plots and findings show that Bayesian inference has the potential to be a valuable tool in the optimization and decision-making processes of PLC maintenance.

Ethical issue

The authors are aware of and complies with best practices in publication ethics, specifically with regard to authorship (avoidance of guest authorship), dual submission, manipulation of figures, competing interests, and compliance with policies on research ethics. The authors adhere to publication requirements that the submitted work is original and has not been published elsewhere.

Data availability statement

Datasets analyzed during the current study are available and can be given following a reasonable request from the corresponding author.

Conflict of interest

The authors declare no potential conflict of interest.

References

- [1] Brown, M. and Proschan, F. (1983) 'Imperfect repair', J Appl Probab, 20, pp. 851–859.
- [2] Cho, D. and Parlar, M. (1991) 'A survey of maintenance models for multi-unit systems', Eur J Oper Res, 51, pp. 1–23. Available at: https://doi.org/10.1016/0377-2217(91)90141-h.
- [3] Nicolai, R. and Dekker, R. (2007) Optimal maintenance of multi_component systems: a review. Complex system maintenance handbook. London: Springer. Available at: https://doi.org/10.1007/978-1-84800-011-7_11.

- Pham, H. and Wang, H. (1996) 'Imperfect maintenance', European Journal of Operational Research, 94(3), pp. 425–438. Available at: https://doi.org/10.1016/S0377-2217(96)00099-9.
- [5] Wang, H. (2002) 'A survey of maintenance policies of deteriorating systems', Eur J Oper Res, 139, pp. 469– 489. Available at: https://doi.org/10.1016/s0377-2217 (01)00197-7.
- [6] Dekker, R. (1995) 'On the use of operations research models for maintenance decision making', Microelectronics Reliability, 35(9–10), pp. 1321–1331. Available at: https://doi.org/10.1016/0026-2714(95)99380-2.
- [7] Marais, K. and Saleh, J. (2009) 'Beyond its cost, the value of maintenance: an analytical framework for capturing its net present value', Reliab Eng Syst Saf, 94, pp. 644–657. Available at: https://doi.org/10.1016/ j.ress.2008.07.004.
- [8] Ming Tan, C. and Raghavan, N. (2008) 'A framework to practical predictive maintenance modeling for multistate systems', 93, pp. 1138–1150. Available at: https://doi.org/10.1016/j.ress.2007.09.003.
- [9] Alizadeh, S. and Sriramula, S. (2018) 'Impact of common cause failure on reliability performance of redundant safety related systems subject to process demand', Reliability Engineering & System Safety, 172, pp. 129–150. Available at: https://doi.org/https://doi.org/10.1016/j.ress.2017.1 2.011.
- [10] Liu, Y. and Frangopol, D.M. (2018) 'Time-dependent reliability assessment of ship structures under progressive and shock deteriorations', Reliability Engineering & System Safety, 173, pp. 116–128. Available at: https://doi.org/https://doi.org/10.1016/j.ress.2018.0 1.009.
- [11] Meng, H., Kloul, L. and Rauzy, A. (2018) 'Modeling patterns for reliability assessment of safety instrumented systems', Reliability Engineering & System Safety, 180, pp. 111–123. Available at: https://doi.org/https://doi.org/10.1016/j.ress.2018.0 6.026.
- [12] Chen, H.-P. and Mehrabani, M.B. (2019) 'Reliability analysis and optimum maintenance of coastal flood defences using probabilistic deterioration modelling', Reliability Engineering & System Safety, 185, pp. 163– 174. Available at: https://doi.org/https://doi.org/10.1016/j.ress.2018.1 2.021.
- [13] Pliego Marugán, A., Peco Chacón, A.M. and García Márquez, F.P. (2019) 'Reliability analysis of detecting false alarms that employ neural networks: A real case study on wind turbines', Reliability Engineering & System Safety, 191, p. 106574. Available at: https://doi.org/https://doi.org/10.1016/j.ress.2019.1 06574.
- [14] Zhu, X., Wang, J. and Yuan, T. (2019) 'Design and maintenance for the data storage system considering system rebuilding process', Reliability Engineering & System Safety, 191, p. 106576. Available at:

https://doi.org/https://doi.org/10.1016/j.ress.2019.1 06576.

 [15] Izquierdo, J., Crespo Márquez, A. and Uribetxebarria, J.
 (2019) 'Dynamic artificial neural network-based reliability considering operational context of assets.', Reliability Engineering &System Safety, 188, pp. 483– 493. Available at: https://doi.org/10.1016/j.ress.2019.0

https://doi.org/https://doi.org/10.1016/j.ress.2019.0 3.054.

- Shi, Y. et al. (2020) 'Condition-based maintenance optimization for multi-component systems subject to a system reliability requirement', Reliability Engineering & System Safety, 202, p. 107042. Available at: https://doi.org/https://doi.org/10.1016/j.ress.2020.1 07042.
- Ma, X. et al. (2020) 'Reliability analysis and conditionbased maintenance optimization for a warm standby cooling system', Reliability Engineering & System Safety, 193, p. 106588. Available at: https://doi.org/https://doi.org/10.1016/j.ress.2019.1 06588.



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(https://creativecommons.org/licenses/by/4.0/).

- [18] Wang, X. et al. (2020) 'Reliability and maintenance for performance-balanced systems operating in a shock environment', Reliability Engineering & System Safety, 195, p. 106705. Available at: https://doi.org/https://doi.org/10.1016/j.ress.2019.1 06705.
- [19] Gao, K. et al. (2020) 'Jointly optimizing lot sizing and maintenance policy for a production system with two failure modes', Reliability Engineering & System Safety, 202, p. 106996. Available at: https://doi.org/https://doi.org/10.1016/j.ress.2020.1 06996.
- [20] Chang, P.-C. et al. (2021) 'Reliability and maintenance models for a time-related multi-state flow network via d-MC approach', Reliability Engineering & System Safety, 216, p. 107962. Available at: https://doi.org/https://doi.org/10.1016/j.ress.2021.1 07962.